Chapter 4
Vertical Alignment

September 2016
CHAPTER 4 VERTICAL ALIGNMENT

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Chapter 4
Vertical Alignment

The roadway vertical alignment plays a significant role in a roadway's safety, aesthetics and project costs. This chapter provides guidance on vertical alignment elements, including: coordination with other design elements, laying out a profile grade line, guidance on grades including maximum and minimum allowable grades, principles and application of vertical curves, and minimum vertical clearances. The Montana Department of Transportation (MDT) Geometric Design Standards provides specific vertical alignment standards relative to a roadway’s functional classification (1). Example calculations and detailed definitions associated with this chapter can be found in Appendix K.

4.1 DESIGN PRINCIPLES AND APPROACH

The design of vertical alignment, similar to horizontal alignment, involves a clear understanding of the design controls and criteria. These include maximum and minimum allowable grades, sight distance, and vertical clearances. In addition, the design team should be aware of how the vertical alignment impacts other design elements and the overall safety and aesthetic appearance of the roadway.

Determination and design of appropriate vertical tangent lengths, gradients, and crest and sag vertical curves form the basis of vertical alignment design. The gradient represents the slope between two adjacent vertical points of intersection and is most commonly expressed as a percentage. Vertical curves used to effect gradual changes between tangent grades consist of two types, sag and crest, and can either be symmetrical or asymmetrical. Exhibit 4-1 illustrates generic crest and sag vertical curves and identifies key elements such as the Tangent Grades ($G_1$ and $G_2$), the Length of Vertical Curve ($L$), the Vertical Point of Curvature (VPC), the Vertical Point of Intersection (VPI), the Vertical Point of Tangency (VPT), and the algebraic difference in grades ($A$).
Minimum lengths of crest or sag curves at various design speeds are commonly defined by the K-value, which represents the horizontal distance needed for a vertical curve to produce a 1-percent change in gradient. Minimum K-values relate lengths of vertical curves to required sight distance on crest curves and to headlight beam distance on sag vertical curves. The design team should consider various options to satisfy sight distance criteria on vertical curves. The design team should also understand that in some instances minimum stopping sight distance can be met without satisfying the minimum K-value.

### 4.1.1 Coordination with Other Design Elements

The design of a vertical alignment must be coordinated with other design elements to ensure a safe, context-sensitive, and cost-effective design. The design elements to be considered when laying out a vertical alignment include:

1. **Consistency.** Use a smooth grade line with gradual changes, consistent with the type of roadway and character of terrain, rather than a line with numerous breaks and short lengths of tangent grades.
2. **Long Grades.** On a long ascending grade, it is preferable to place the steepest grade at the bottom and flatten the grade near the top.
3. **Intersections.** Maintain moderate grades through intersections to facilitate turning movements. See Chapter 6, Section 6.1.3 for specific information on vertical alignment through intersections.
4. **Broken-Back Curvature.** "Broken-back" grade lines (two vertical curves in the same direction separated by a short section of tangent
4.1 Sight Alignment

4.1.1 Grade

grade) should be avoided where feasible. One long vertical curve is more desirable.

5. “Roller Coaster”. A “roller coaster” type of profile may be an outcome of fitting an alignment across varying topography; however the design team should avoid excessive ups and downs, which may be unpleasant aesthetically and difficult for drivers to navigate.

6. **Vertical Point of Intersection (VPI) Locations.** Set VPI locations at even 5-foot stations if practical.

7. **Environmental Impacts.** The vertical alignment should be properly coordinated with impacts to potential environmental resources and other natural resources, for example, encroachment onto wetlands.

8. **Consideration of Natural/Man-Made Features.** The vertical alignment should take into consideration the natural topography, available right-of-way, utilities, roadside development, structures, and natural or man-made drainage features.

9. **Drainage.** See Section 4.2.8 for drainage aspects to consider when establishing a vertical alignment.

4.1.2 Sight Distance Considerations

Two key considerations in the establishment of a vertical alignment are stopping sight distance and passing sight distance.

1. **Stopping Sight Distance.** All vertical curves must be designed to provide at least the stopping sight distance shown in Chapter 2, Exhibits 2-2 or 2-3. Additional stopping sight distance should be provided when practical. Determining the minimum length of a crest vertical curve using stopping sight-distance criteria typically results in a curve that is satisfactory from a safety, comfort, and appearance standpoint. Stopping sight distance for sag vertical curves at an undercrossing should also be considered. Outside of this condition, sight distance for sag vertical curves is provided as long as the design allows the vehicular headlights to illuminate the roadway surface. Sight distance on a roadway through a grade separation should be at least as long as the minimum stopping sight distance and where practical, even longer. Sections 4.4.1 and 4.4.2 and Chapter 2, Section 2.8.1.2 provide additional information for determining stopping sight distance for vertical curves.

2. **Passing Sight Distance.** At some locations, it may be desirable to provide passing sight distance in the design of crest vertical curves. On rural reconstruction projects, the design team should attempt to provide passing sight distance over as much of the roadway length as practical. It will generally not be cost-effective, however, to make significant improvements to the horizontal and vertical alignment solely to increase the available passing sight distance. Chapter 2, Section 2.8.3 discusses the application and design values for passing sight distance.
4.1.3 Coordination of Horizontal and Vertical Alignment

Horizontal and vertical alignments should be designed in coordination, especially for projects on a new alignment. The design team should carefully evaluate the interdependence of these two roadway design features to enhance the roadway's safety and improve its operation. The following should be considered in the coordination of horizontal and vertical alignment:

1. **Balance.** The horizontal curvature and vertical grades and curves should be in proper balance. Over-emphasizing one or the other may lead to an unbalanced design. A compromise between the two extremes typically produces the best design relative to safety, operations, ease of driving, and uniformity of operations and aesthetics.

2. **Coordination.** Locating vertical and horizontal points of intersection (PIs) at approximately the same station generally results in a more pleasing appearance and reduces the number of sight distance restrictions. Successive changes in profile that are aligned with similar changes in horizontal curvature results in a series of humps visible to the driver for some distance, which may produce an unattractive design. However, coordination of the horizontal and vertical alignment should be tempered under conditions for crest and sag vertical curves described as follows.

   a. **Crest Vertical Curves.** Sharp horizontal curvature should not be introduced at or near the top of pronounced crest vertical curves. This is undesirable because the driver cannot perceive the horizontal change in alignment, especially at night when headlight beams project straight ahead into space. This problem can be avoided if the horizontal curvature occurs prior to the vertical curvature or by using curve design values exceeding minimums.

   b. **Sag Vertical Curves.** Sharp horizontal curves should not be introduced at or near the low point of pronounced sag vertical curves or at the bottom of steep vertical grades. Because visibility to the road ahead is foreshortened, only flat horizontal curvature will avoid an undesirable, distorted appearance. At the bottom of long grades, vehicular speeds often are higher, particularly for trucks, and erratic operations may occur, especially at night.

3. **Passing Sight Distance.** In some cases, the need for frequent passing opportunities and a higher percentage of passing sight distance may supersede the desirability of combining horizontal and vertical alignment. In these cases, it may be necessary to provide long tangent sections to ensure sufficient passing sight distance.

4. **Intersections.** At intersections, horizontal and vertical alignments should be as flat as practical to provide sufficient sight distance and gradients for vehicles to slow or stop and to provide accessible cross slopes for people walking or using mobility aids.
5. **Divided Highways.** On divided highways with wide medians, it is typically advantageous to provide independent alignments for the roadway on each side of the median.

6. **Residential Areas.** Design the alignment to minimize nuisance factors, such as noise and visual pollution, to neighborhoods. Minor adjustments to the horizontal or vertical alignment or both may increase the buffer zone between the roadway and residential areas.

7. **Access.** Existing and proposed public and private approaches should be evaluated when coordinating horizontal and vertical alignment to ensure minimum intersection sight distance requirements are met.

8. **Drainage.** See Section 4.2.8 for drainage aspects to consider when coordinating horizontal and vertical alignment.

9. **Aesthetics.** The design team should always be mindful of, and work within, existing topographic and infrastructure constraints when establishing alignments. Where feasible and other constraints are not present, design the alignment to enhance attractive scenic views of rivers, rock formations, parks, or other features.

### 4.2 PROFILE GRADE LINE

The profile grade line is defined by a series of tangents connected by parabolic vertical curves, as needed. The profile grade line impacts a roadway’s costs, aesthetics, safety, and operations. Design considerations for establishing the profile grade line are discussed below.

#### 4.2.1 Profile Grade Line Locations

The location of the profile grade line on the roadway cross section varies according to the roadway type, whether or not the roadway is divided or undivided, and by the median type. The profile grade line locations are shown in the typical cross section exhibits provided in Chapter 5, Section 5.6. The recommended locations for profile grade lines and axes of rotation for various typical sections are provided below.

1. **Roadways with Narrow Medians (≤ 10 feet wide).** For roadways with narrow medians (flush or raised), there is typically only one profile grade line at the centerline of the median or traveled way. A single profile grade line may be used for medians up to 15 feet in width if appropriate given the context. Typically, the superelevation axis of rotation is at the profile grade line at the centerline.

2. **Roadways with Medians ≥ 10 feet to 76 feet wide.** For roadways with medians (flush, raised, or depressed) in this range of widths, a profile grade line should typically be provided for each roadway and be established on the median edge of pavement. Typically, the superelevation axes of rotation are at the median edge of pavement profile grade lines.

3. **Roadways with Medians > 76 feet wide.** Roadways with wide medians typically use independent alignments for each direction of travel and are

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**Stakeholder outreach early in the project development process can help identify critical features that should be considered throughout the design.**

**Exceptions to the location of the superelevation axis of rotation may be necessary to better fit the existing topography and other constraints and/or to provide for improved drainage.**
treated as two separate roadways. Therefore, a profile grade line is necessary for each roadway and is typically provided at the centerline of each traveled way. Typically, the superelevation axes of rotation are at the centerline profile grade lines.

4. **Two-Lane Rural Highways.** For two-lane rural highways, there is typically only one profile grade line at the centerline of the traveled way. Typically, the superelevation axis of rotation is at the profile grade line at the centerline.

5. **Curbed Roadways.** For curbed roadways, follow the guidance provided in items 1-4 above depending on the median condition. In addition, separate profiles at the top back of curb should be provided where necessary to ensure positive drainage and to match existing development.

### 4.2.2 Urban Grade Design

Laying out profile grade lines in urban areas often is more challenging due to limited right-of-way, closely spaced intersections, the need to meet existing roadside development, and drainage. The following provides several considerations that should be reviewed when developing a profile grade line on an urban project:

1. **Vertical Curves.** Long vertical curves on urban roadways are generally impractical. The design team will typically need to lay out the profile grade line to meet existing conditions. Therefore, no minimum vertical curve lengths are specified for urban roadways; however, the design team must ensure minimum sight distance criteria are satisfied and that design principles outlined in Section 4.1 are taken into consideration. Where practical, VPIs should be located at or near the centerlines of cross streets. In addition, at signalized and stop-controlled intersections some flattening of the approaches may be desired to allow vehicles to more easily pull away from a stopped position and to provide accessible cross slopes for people walking or using mobility aids.

2. **Drainage.** Urban roadways will often have curbs, which may complicate the layout of the profile grade to facilitate drainage. Reference Section 4.2.8 for additional guidance on drainage considerations when establishing profile grade lines on urban roadways.

3. **Spline Curves.** A spline curve is a curve connecting specific points, such as elevation points. Spline curves can be helpful in establishing profile grade lines in urban areas where it is necessary to meet numerous elevation restrictions in relatively short distances. Spline curves may also be used when developing profiles for the top of curbs. Show elevations along spline curves at 10-foot intervals.

4. **Earthwork Balance.** In general, balancing of earthwork is typically impractical in urban areas. An excess of excavation is preferable to the need for borrow due to the generally higher cost of borrow in urban areas.

Chapter 8, Section 8.4 provides additional design considerations for urban areas.
5. **Limited Right-of-Way.** Careful consideration should be given when substantially lowering or raising the profile grade line. This can result in more right-of-way impacts and access issues (for intersecting roadways and adjacent property) along urban roadways.

### 4.2.3 Earthwork Balance/Soil Conditions

Where practical and consistent with other project objectives, design the profile grade line to provide a balance of earthwork. This should not be achieved, however, in a way that inhibits smooth grade lines and sight distance requirements at vertical curves. Ultimately, a project-by-project assessment will determine whether a project will require borrow, have excess material, or be balanced.

The type of earth material encountered often influences the profile grade line at certain locations. If rock is encountered, for example, it may be more economical to raise the grade and reduce the rock excavation. Soils which are unsatisfactory for embankment or cause a stability problem in cut areas may also be determining factors in establishing a grade line. The presence of poor quality or saturated soils may also influence the profile grade line due to the extra expense associated with excavating or stabilizing these soils. In these situations, the design team should consider soil information from soil borings and cores in development of the profile grade.

### 4.2.4 Field Constraints

Special grade control may be necessary across flats, lake beds, sloughs, creek bottoms, important intersecting roads, in front of improved property, at places subject to snow drifting, and at other places requiring special attention. The design team should review and consider these recommendations when establishing the profile grade line.

### 4.2.5 Ties with Existing Roadways/Adjoining Projects

A smooth transition is needed between the proposed profile grade line of the project and the existing grade line of an adjacent roadway section, or in some cases the proposed grade line for an adjacent project. Based on road classification and project context, profile grade lines should be reviewed beyond the beginning and end of a project to ensure adequate sight distance and proper conformance with existing profile grades. Connections to existing roadway approaches should be compatible with the design speed of the new project and fit the overall context of the corridor.

### 4.2.6 Transition Grades

Transition grades are used where a roadway alignment diverges from or merges into the mainline alignment; examples include freeway ramps, turning roadways, and detours. Exhibit 4-2 illustrates the location of the transition grade for ramps and detours. The transition grade is the profile grade line where the taper from the mainline begins (section “A” on Exhibit 4-2) to the point where the ramp or roadway grade becomes independent of the mainline (for example, at the gore nose, section “D” on Exhibit 4-2).
The transition grade is dependent on the mainline grade and cross slopes until the ramp or roadway becomes independent (Section "D" in Exhibit 4-2). For ramps, the transition grade should extend from the beginning of the taper of the ramp away from the mainline to a point where the distance between the outside shoulder of the mainline and the inside shoulder of the ramp is 8 feet. For detours, the transition grade should extend from the beginning of the taper from the mainline to a point where the distance between the outside shoulder of the mainline and the inside shoulder of the detour is 4 feet. Chapter 10 provides additional design consideration details for detours.

Site constraints may dictate the necessity for cross slope superelevation transition within the length of the transition grade. Also, ramps (and other turning roadways) that intersect mainline in superelevated sections, may require different cross slopes due to opposite directions of curvature or for difference in design speeds. For these cases, the cross slope of the ramp in the transition grade area may vary from the cross slope of the mainline to permit a transition to the appropriate superelevation/cross slope of the ramp. When the cross slope is different from that of the mainline, the transition grade is established through a spline curve (best fit curve) through elevations determined and shown at intervals of 10 to 20 feet. Developing separate spline curves for each edge of the ramp traveled way and the gore area is helpful in ensuring proper design and construction is achieved. The point elevations defining the spline curve should be determined with consideration to the following:

1. The grade and cross slope of the turning roadway should transition smoothly without abrupt changes.
2. The maximum relative longitudinal gradient criteria, adjusted for width should be met. See Section 3.3.4.
3. Drainage should be adequate to avoid ponding throughout the paved portion of the gore or spread width encroaching onto the traveled way.
4. The maximum algebraic difference in cross slope at the crossover crown line (points A to B) are limited as shown in Exhibit 4-3. Additional information can be found in the AASHTO Green Book (2).
### 4.2.7 Bridges and Drainage Structures

The design of profile grade lines should be carefully coordinated with any bridges or drainage structures within the project limits. The following will apply:

1. **Vertical Clearances.** Section 4.5 provides additional information on minimum vertical clearances. The criteria in Section 4.5 must be met where a new roadway and structure will be constructed over an existing roadway or where the roadway will be reconstructed under an existing bridge. When developing the preliminary profile grade line, an important element in determining available vertical clearance is the assumed structure depth. This will be based on the structure type, span lengths, and depth-to-span ratio. The design team should coordinate with the Bridge Bureau to obtain an estimated depth of structure or minimum profile elevation.

2. **Roadway Under Bridge.** Where practical, the low point of a roadway sag vertical curve under a bridge should not be in close proximity to the bridge. This will help minimize ice accumulations near and under the bridge, and it will reduce the ponding of water which may weaken the earth foundation beneath the bridge. To achieve these objectives, the low point of a roadway sag curve should be approximately 100 feet or more from the bridge.

3. **Bridges Over Water.** Where the proposed roadway will cross bodies of water, the bridge elevation must be consistent with the necessary waterway opening to meet MDT's hydraulic requirements. The design team provides the Bridge Bureau with the preliminary profile grade line and proposed cross section. The Bridge Bureau determines minimum bridge elevations based on the hydraulic requirements and structure type. Establishing grades for structures over drainage features can be an iterative process, requiring coordination between the hydraulic, roadway, and bridge design teams.

4. **Bridges Over Railroads.** Any proposed roadways over railroads must meet the applicable criteria (e.g., vertical clearances, structure type and depth). See Section 4.5 for minimum vertical clearance criteria. The design team should contact the Bridge Bureau and the Utilities Section for more information.

5. **High Embankments.** The design team should consider the impacts of high embankments on structure lengths. A roadway alignment that creates high embankments will increase the span length, thus increasing structure costs.

6. **Bridge Decks.** Bridge deck drainage is often less efficient than the drainage on roadway sections and the spread of water on the deck is
often more difficult to control. This is primarily due to deck cross slopes that are often flatter, issues with debris build-up on the deck, and the inherent limitations of inlets placed on a bridge deck. Zero or relatively flat gradients and locating the low point of a sag vertical curve on the bridge deck should be avoided where feasible.

4.2.8 Drainage

The profile grade line should be compatible with the roadway drainage design. The design team should consider the following in the establishment of a profile grade line:

1. **Existing Drainage Patterns.** Strive to minimize interference with existing drainage patterns to the extent possible. If the redirection of existing flows is unavoidable, careful attention and consideration should be given to the impacts this may have on adjacent properties, flood control measures, and water quality control.

2. **Minimum Longitudinal Grades.** The profile grade line is important to consider on any roadway, but especially for shallow grades along curved roadways.
   a. **Curbed Roadways.** The centerline profile on roadways with curbs should have a minimum longitudinal gradient of 0.5 percent where feasible. Longitudinal gradients less than 0.5 percent can be considered under special circumstances, but may require additional storm drain inlets or other special drainage treatments. Because surface drainage is retained within the roadway, special care should be taken to avoid flat spots where water may pond. Separate profile grade lines at the top back of curb should be developed when needed to ensure adequate drainage to inlets. Curb elevations should be designed to permit drainage flow into the gutter and to avoid ponding of water.
   b. **Shouldered Roadways.** Where feasible, a 0.5-percent minimum longitudinal grade should be provided to encourage drainage and avoid ponding in the ditches away from the planned low points. Longitudinal gradients that are level may be acceptable on pavements in fill areas with an adequate roadway crown to drain laterally. The minimum longitudinal grade in cut sections is 0.2 percent. Independent ditch grades should be provided to ensure adequate roadside drainage when minimum roadway grades cannot be provided.

3. **Vertical Curves.** Drainage should be considered in the design of vertical curves, especially where curbed sections are used. Drainage problems should not be experienced if the vertical curvature is pronounced enough that a minimum longitudinal grade of at least 0.3 percent is reached at a point about 50 feet from either side of the high or low point of the curve. To ensure this objective is achieved, the length of the vertical curve should be based upon a maximum K-value of 167 or less. For vertical curves on curbed sections where this K-value is exceeded, the drainage design should be more carefully evaluated near the high or
low point of the curve. Separate curb profiles resulting in varying cross slopes may be necessary to ensure adequate drainage and for sag vertical curves it may be necessary to install flanking inlets on either side of the low point.

4. **Intersections.** At intersections, the surface drainage should preferably be intercepted upstream of the intersection. Where surface drainage is provided across intersecting roadways, use of valley gutters should be considered.

5. **Culverts.** The roadway elevation should provide at least the minimum cover indicated in the culvert fill height tables in Chapter 11, Section 11.2. The top of culvert should be located below the top of the subgrade where feasible. The low points of a sag vertical curve should not be located directly above culverts where feasible. In the event that flooding overtops the roadway, culverts located at low points have a greater chance of being washed out. See Chapter 11 for additional information on culvert designs.

### 4.2.9 Snow Drifting

The profile grade line should minimize snow drift challenges. Where practical, the profile grade line should be at least 2 feet above the natural ground level on the windward side of the roadway to prevent snow from drifting onto the roadway and to promote snow blowing off the roadway. For cut sections, the use of wider ditches, flatter than standard backslopes, other drifting mitigation features (e.g., snow fence), or combinations thereof may be appropriate.

### 4.2.10 Erosion Control

To minimize erosion, the design team should consider the following relative to the grade line:

- Minimize the number of deep cuts and high fill sections;
- Conform to the contour and drainage patterns of the area;
- Make use of natural land barriers and contours to divert runoff and confine erosion and sedimentation;
- Minimize the amount of disturbance;
- Make use of existing vegetation;
- Reduce slope length and steepness to attempt to contain erosion within the right-of-way and not deposit sediment on or erode away adjacent land;
- Avoid locations having high erosion potential; and
- Avoid cut or fill sections in seepage areas, including areas with springs or a high water table.

### 4.2.11 Project Types

A new profile grade line should be shown for reconstruction projects, pavement pulverization projects, and projects that involve major surfacing rehabilitation. It will likely not be necessary to provide profile grade lines for pavement preservation or other minor surfacing rehabilitation projects.
4.3 GRADES

The use of maximum and minimum grades in a vertical alignment has an impact on the earthwork, sight distance, drainage, and other design related items. The critical length of grade and its impact on the vertical alignment design is also an important consideration.

4.3.1 Maximum Grades

The MDT Geometric Design Standards document presents criteria for maximum grades based on functional classification, type of terrain, and in some cases, design speed (1). The maximum grades should be used only where necessary. Where practical, use grades flatter than the maximum.

4.3.2 Minimum Grades

Minimum grades are primarily determined based on drainage needs and requirements and are described in detail in Section 4.2.8.

4.3.3 Critical Length of Grade and Truck Climbing Lanes

The critical length of grade is the maximum length of a specific upgrade on which a loaded truck can operate without experiencing a specified reduction in speed. The roadway gradient in combination with the length of grade will determine the truck speed reduction on upgrades.

The Traffic and Safety Bureau will typically determine the need for truck-climbing lanes and will provide the design details for these lanes where they are warranted.

4.4 VERTICAL CURVES

Vertical curves should be simple in application and should result in a design enabling the driver to see the road ahead, enhancing vehicle control, pleasing in appearance, and adequate for drainage. On urban roadways, vertical curves are not required when the algebraic difference in grades is less than 1 percent. However, the use of vertical curves on urban roadways should be considered when the algebraic difference in grades is greater than 0.5 percent. Vertical grade breaks without curves are not allowed on rural roadways.

If a vertical curve is justified, the following sections outline criteria for laying out both crest and sag vertical curves. The discussion centers around the principle of providing adequate sight distance, which is commonly applied through a K-value, defined as the horizontal distance needed to produce a 1-percent change in grade. The design team should recognize that adequate sight distance will be provided when a vertical curve is designed to meet or exceed the minimum K-value for a given design speed. However, adequate sight distance may still be provided even if the minimum K-value is not satisfied; this is particularly true for longer sight distance values. The design team can check sight distance on a plotted profile if in doubt.
Crest vertical curves are in the shape of a parabola. The basic equations for determining the minimum length of a crest vertical curve are:

When $S$ is less than $L$,

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2}$$

When $S$ is greater than $L$,

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

Where:

- $L$ = length of vertical curve, feet
- $A$ = algebraic difference between the two tangent grades, percent
- $S$ = sight distance, feet
- $h_1$ = height of eye above road surface, feet
- $h_2$ = height of object above road surface, feet

The length of the crest vertical curve will depend upon the algebraic difference between two tangent grades for the specific curve, the selected sight distance, the height of eye, and the height of object. The following sections discuss the selection of these values. Equations 4.4-1 and 4.4-2 allow the design team to input different values for $h_1$ and $h_2$ to determine the length of vertical curve ($L$) based on the algebraic difference in grades ($A$) and required sight distance ($S$).

The application of Equations 4.4-1 and 4.4-2 may be an iterative process, as described below, depending on the calculated value of $L$.

1. Calculate an initial value of $L$ based on Equation 4.4-1 and compare this value to the required value of $S$.
2. If $S$ is less than $L$, then use the calculated value of $L$ from Equation 4.4-1 for designing the crest vertical curve.
3. If $S$ is greater than $L$, then recalculate $L$ based on Equation 4.4-2 and use this recalculated value of $L$ for designing the crest vertical curve.
4. For small values of $A$, calculated vertical curve lengths may be near zero because the sight line passes over the high point of the curve. In these cases, use Equation 4.4-5 to determine the minimum length of vertical curve.

Equations 4.4-1 and 4.4-2 can be reduced to Equation 4.4-3 by assuming a required sight distance and specific values for $h_1$ and $h_2$ based on the type of sight distance required (such as stopping sight distance or passing sight distance).
\[ L = KA \]

Where:
- \( L \) = length of vertical curve, feet
- \( K \) = horizontal distance needed to produce a 1-percent change in gradient
- \( A \) = algebraic difference between the two tangent grades, percent

Determining \( K \)-values for crest vertical curves is described in detail in the following sections. For design purposes, the calculated length of curve based on the rounded \( K \)-value should be rounded up to the next highest 50-foot increment where practical. Rounding to the next highest 10-foot increment is typically appropriate for urban roadways.

### 4.4.1.1 Minimum Stopping Sight Distance

Equation 4.4-4 assumes the typical values for \( h_1 \) and \( h_2 \), 3.5 feet and 2.0 feet, respectively, to calculate the \( K \)-value for crest vertical curves at different required sight distances. The values of \( K \) derived from Equation 4.4-4 (when \( S \) is less than \( L \)) also can be used without significant error where \( S \) is greater than \( L \).

\[ K = \frac{S^2}{2158} \]

Where:
- \( K \) = horizontal distance needed to produce a 1-percent change in gradient
- \( S \) = sight distance, feet

The principal control in the design of crest vertical curves is to ensure that, at a minimum, stopping sight distance is provided throughout the curve. Short vertical curves are sometimes unavoidable in urban settings and may not meet minimum \( K \)-values, but may meet the required stopping sight distance if checked on a plotted profile.

Exhibit 4-4 presents minimum stopping sight distances for various downgrade conditions and the associated \( K \)-values for passenger cars on crest vertical curves. The values in Exhibit 4-4 are calculated using Equation 4.4-4. The minimum values represent the lowest acceptable sight distance on a roadway at a given downgrade; however, the design team should provide a design in which the \( K \)-values meet the greatest practical stopping sight distance.
<table>
<thead>
<tr>
<th>Design Speed (V) (mph)</th>
<th>Stopping Sight Distances (ft)</th>
<th>K-Values (^{(1)}) ((K=S^2/2158))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level (^{(2)}) (&lt;3%)</td>
<td>Downgrades</td>
</tr>
<tr>
<td></td>
<td>3% 6% 9%</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>80 80 82 85</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>115 116 120 126</td>
<td>7</td>
</tr>
<tr>
<td>25</td>
<td>155 158 165 173</td>
<td>12</td>
</tr>
<tr>
<td>30</td>
<td>200 205 215 227</td>
<td>19</td>
</tr>
<tr>
<td>35</td>
<td>250 257 271 287</td>
<td>29</td>
</tr>
<tr>
<td>40</td>
<td>305 315 333 354</td>
<td>44</td>
</tr>
<tr>
<td>45</td>
<td>360 378 400 427</td>
<td>61</td>
</tr>
<tr>
<td>50</td>
<td>425 446 474 507</td>
<td>84</td>
</tr>
<tr>
<td>55</td>
<td>495 520 553 593</td>
<td>114</td>
</tr>
<tr>
<td>60</td>
<td>570 598 638 686</td>
<td>151</td>
</tr>
<tr>
<td>65</td>
<td>645 682 728 785</td>
<td>193</td>
</tr>
<tr>
<td>70</td>
<td>730 771 825 891</td>
<td>247</td>
</tr>
<tr>
<td>75</td>
<td>820 866 927 1003</td>
<td>312</td>
</tr>
<tr>
<td>80</td>
<td>910 965 1035 1121</td>
<td>384</td>
</tr>
</tbody>
</table>

(1) K-values are calculated using rounded stopping sight distances, eye height of 3.5 feet and object height of 2 feet: \(K=S^2/200(\sqrt{3.5}+\sqrt{2})\). The values of K derived from this equation (when \(S\) is less than \(L\)) also can be used without significant error where \(S\) is greater than \(L\).

(2) The stopping sight distance for the “Level (<3%)” grades also applies to upgrades.

### 4.4.1.2 Minimum Length of Curve (Rural Roadways)

Where the algebraic difference in grades \((A)\) is small, the calculated curve lengths may be near zero (based on Equations 4.4-1 and 4.4-2). However, angle points are not allowed on rural roadways. Therefore, the minimum length of crest vertical curve is based on Equation 4.4-5.

\[
L_{\text{min}} = 3V
\]

Where:

- \(L_{\text{min}}\) = minimum length of vertical curve, feet
- \(V\) = design speed, mph

Note: Refer to Equations 2.8-3 and 4.4-4 for calculating stopping sight distances and K-values for downgrades other than those shown in Exhibit 4-4. Interpolation between the values shown is not appropriate.

The steeper of the two gradients on either side of the crest vertical curve should be used in establishing the minimum stopping sight distance and K-value, except in the case of one-way roadways where the gradient in the direction of travel should be used.

MDT does not suggest a minimum length of crest vertical curve for urban roadways.

**Equation 4.4-5**

\[
L_{\text{min}} = 3V
\]
4.4.1.3 Passing Sight Distance

At some locations, it may be desirable to provide passing sight distance in the design of crest vertical curves. On rural reconstruction projects, the design team should attempt to provide passing sight distance over as much of the roadway length as practical. However, making significant improvements to the horizontal and vertical alignments solely to increase the available passing sight distance will generally not be cost effective. Chapter 2, Section 2.8.3 discusses the application and design values for passing sight distance. These values of $S$ are used in the basic equations for crest vertical curves (Equations 4.4-1 and 4.4-2). Equation 4.4-6 assumes the height of eye ($h_1$) is 3.5 feet, and the height of object ($h_2$) is also 3.5 feet. The values of $K$ derived from Equation 4.4-6 (when $S$ is less than $L$) also can be used without significant error where $S$ is greater than $L$.

$$K = \frac{S^2}{2800}$$

Where:

$K$ = horizontal distance needed to produce a 1-percent change in gradient  
$S$ = sight distance, feet

Exhibit 4-5 presents the minimum passing sight distances and the associated $K$-values for passenger cars on crest vertical curves.

<table>
<thead>
<tr>
<th>Design Speed (mph)</th>
<th>Minimum Passing Sight Distance For Design (ft)</th>
<th>Calculated $K$-Values (Equation 4.4-6)</th>
<th>$K$-Values Rounded For Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>400</td>
<td>57.1</td>
<td>57</td>
</tr>
<tr>
<td>25</td>
<td>450</td>
<td>72.3</td>
<td>72</td>
</tr>
<tr>
<td>30</td>
<td>500</td>
<td>89.3</td>
<td>89</td>
</tr>
<tr>
<td>35</td>
<td>550</td>
<td>108.0</td>
<td>108</td>
</tr>
<tr>
<td>40</td>
<td>600</td>
<td>128.6</td>
<td>129</td>
</tr>
<tr>
<td>45</td>
<td>700</td>
<td>175.0</td>
<td>175</td>
</tr>
<tr>
<td>50</td>
<td>800</td>
<td>228.6</td>
<td>229</td>
</tr>
<tr>
<td>55</td>
<td>900</td>
<td>289.3</td>
<td>289</td>
</tr>
<tr>
<td>60</td>
<td>1000</td>
<td>357.1</td>
<td>357</td>
</tr>
<tr>
<td>65</td>
<td>1100</td>
<td>432.1</td>
<td>432</td>
</tr>
<tr>
<td>70</td>
<td>1200</td>
<td>514.3</td>
<td>514</td>
</tr>
<tr>
<td>75</td>
<td>1300</td>
<td>603.6</td>
<td>604</td>
</tr>
<tr>
<td>80</td>
<td>1400</td>
<td>700.0</td>
<td>700</td>
</tr>
</tbody>
</table>

(1) Passing sight distances are from Chapter 2, Section 2.8.3.
(2) $K$-values are calculated using the passing sight distance, eye height of 3.5 feet, and object height of 3.5 feet: $K = \frac{S^2}{2800(\sqrt{3.5} + \sqrt{3.5})^2}$. The values of $K$ derived from this equation (when $S$ is less than $L$) also can be used without significant error where $S$ is greater than $L$. 

Exhibit 4-5 Minimum K-values for Crest Vertical Curves - Passing Sight Distance
4.4.2 Sag Vertical Curves

Sag vertical curves are also in the shape of a parabola. Typically, they are designed to allow the vehicular headlights to illuminate the roadway surface (i.e., the height of vehicular headlights is 2.0 feet and the height of object is 0.0 feet over a given distance \( S \)). These assumptions yield the following basic equations for determining the minimum length of sag vertical curves:

When \( S \) is less than \( L \),

\[
L = \frac{AS^2}{200h_3 + 3.5S}
\]

When \( S \) is greater than \( L \),

\[
L = 2S - \frac{200h_3 + 3.5S}{A}
\]

Where:

\( L \) = length of vertical curve, feet  
\( A \) = algebraic difference between the two tangent grades, percent  
\( S \) = sight distance, feet  
\( h_3 \) = height of headlights above pavement surface, feet

The length of the sag vertical curve will depend upon the algebraic difference between two tangent grades for the specific curve, the selected sight distance, and headlight height. Equations 4.4-7 and 4.4-8 allow the design team to input a different value for \( h_3 \) to determine the length of vertical curve (\( L \)) based on the algebraic difference in grades (\( A \)) and required sight distance (\( S \)).

The application of Equations 4.4-7 and 4.4-8 may be an iterative process, as described below, depending on the calculated value of \( L \).

1. Calculate an initial value of \( L \) based on Equation 4.4-7 and compare this value to the required value of \( S \).
2. If \( S \) is less than \( L \), use the calculated value of \( L \) from Equation 4.4-7 for designing the sag vertical curve.
3. If \( S \) is greater than \( L \), recalculate \( L \) based on Equation 4.4-8 and use this recalculated value of \( L \) for designing the sag vertical curve.
4. For small values of \( A \), calculated vertical curve lengths may be near zero. In these cases, use Equation 4.4-5 to determine the minimum length of vertical curve.

Equations 4.4-7 and 4.4-8 can be reduced to Equation 4.4-9 by assuming a required sight distance and a specific value for \( h_3 \) (headlight height).

\[
L = KA
\]

Where:

\( L \) = length of vertical curve, feet  
\( K \) = horizontal distance needed to produce a 1-percent change in gradient  
\( A \) = algebraic difference between the two tangent grades, percent

Equation 4.4-7

Equation 4.4-8

Note: For small values of \( A \), calculated vertical curve lengths may be near zero. In these cases, use Equation 4.4-5 to determine the minimum length of vertical curve.

A basic assumption in the application of Equations 4.4-7 and 4.4-8 is that only one curve occurs within the required sight distance. Refer to Appendix F for discussion on sight distance influenced by multiple curves.

Equation 4.4-9
Determining K-values for sag vertical curves is described in detail in the following section. For design purposes, the calculated length of curve based on the rounded K-value should be rounded up to the next highest 50-foot increment where practical. Rounding to the next highest 10-foot increment is typically appropriate for urban roadways.

4.4.2.1 Minimum Stopping Sight Distance

Equation 4.4-10 assumes the typical value for $h_3$, 2.0 feet, to calculate the K-value for sag vertical curves at different required sight distances. The values of K derived from Equation 4.4-10 (when $S$ is less than $L$) also can be used without significant error where $S$ is greater than $L$.

$$K = \frac{S^2}{400 + 3.5S}$$

Where:

- $K$ = horizontal distance needed to produce a 1-percent change in gradient
- $S$ = sight distance, feet

The principal control in the design of sag vertical curves is to consider if, at a minimum, stopping sight distance (SSD) is provided for headlight illumination throughout the curve. The design assumes that there is a 1.0-degree upward divergence of the light beam from the longitudinal axis of the headlights.

Exhibit 4-6 presents minimum stopping sight distances for various downgrade conditions and the associated K-values for passenger cars on sag vertical curves. The values in Exhibit 4-6 are calculated using Equation 4.4-10. The minimum values represent the lowest acceptable sight distance on a roadway at a given downgrade; however, the design team should consider providing a design in which the K-values meet the greatest practical stopping sight distance.
### Design Speed (V) (mph)

<table>
<thead>
<tr>
<th>Design Speed (V)</th>
<th>Stopping Sight Distances (ft)</th>
<th>K-Values&lt;sup&gt;(1)&lt;/sup&gt; (K=S&lt;sup&gt;2&lt;/sup&gt;/(400+3.5S))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level (&lt;3%)</td>
<td>Downgrades</td>
</tr>
<tr>
<td></td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>15</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>20</td>
<td>115</td>
<td>116</td>
</tr>
<tr>
<td>25</td>
<td>155</td>
<td>158</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>205</td>
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<tr>
<td>35</td>
<td>250</td>
<td>257</td>
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<td>40</td>
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<td>45</td>
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<td>446</td>
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<td>60</td>
<td>570</td>
<td>598</td>
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<tr>
<td>65</td>
<td>645</td>
<td>682</td>
</tr>
<tr>
<td>70</td>
<td>730</td>
<td>771</td>
</tr>
<tr>
<td>75</td>
<td>820</td>
<td>866</td>
</tr>
<tr>
<td>80</td>
<td>910</td>
<td>965</td>
</tr>
</tbody>
</table>

<sup>(1)</sup> K-values calculated using rounded stopping sight distances and a headlight height (h<sub>3</sub>) of 2 feet: K=S<sup>2</sup>/(200*2.0+3.5S). The values of K derived from this equation (when S is less than L) also can be used without significant error where S is greater than L.

### 4.4.2.2 Minimum Length of Curve (Rural Roadways)

For most sag vertical curves on rural roadways, the minimum length of curve should be based on Equation 4.4-5 (L<sub>min</sub> = 3V).

### 4.4.3 Asymmetrical Vertical Curves

Occasionally, it is necessary to use an asymmetrical vertical curve to obtain clearance on a structure or to meet other field conditions. This curve is similar to the parabolic vertical curve, except the curve is not symmetrical about the VPI (different curve lengths on each side of the VPI). See Appendix K for examples of the application and calculation of asymmetrical vertical curves.

### 4.4.4 Vertical Curve Through a Fixed Point

A vertical curve sometimes must be designed to pass through an established point. For example, it may be necessary to tie into an existing transverse road or to clear existing structures. See Appendix K for examples of the application and calculation of a vertical curve through a fixed point.
4.5 MINIMUM VERTICAL CLEARANCES

Exhibit 4-7 summarizes the minimum vertical clearances for various roadway classifications and conditions passing under a crossing road or bridge structure.

<table>
<thead>
<tr>
<th>Type</th>
<th>Minimum Clearance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway under Roadway</td>
<td>17.0 (1)</td>
</tr>
<tr>
<td>Principal and Minor Arterial under Roadway</td>
<td>17.0 (1)</td>
</tr>
<tr>
<td>Collector under Roadway</td>
<td>16.5 (1)</td>
</tr>
<tr>
<td>Rural Local under Roadway</td>
<td>14.5 (1)</td>
</tr>
<tr>
<td>Roadway under Pedestrian Bridge</td>
<td>Varies (2)</td>
</tr>
<tr>
<td>Roadway under Traffic Signal</td>
<td>17.0 (1)(3)</td>
</tr>
<tr>
<td>Railroad under Roadway (Typical)</td>
<td>23.29 (4)</td>
</tr>
<tr>
<td>Roadway under Sign Truss</td>
<td>17.0 (1)</td>
</tr>
</tbody>
</table>

(1) Value allows 0.50 feet for future resurfacing.
(2) The vertical clearance should be the same as for the roadway under a highway bridge for the designated type of roadway.
(3) Distance is measured from roadway surface to the bottom of the signal at the bottom of the backplate.
(4) Measured from the top of the highest rail. Crossings over switch yards should provide a minimum of 26 feet of vertical clearance.

4.6 REFERENCES