

**BEFORE-AFTER CRASH ANALYSIS:
A PRIMER FOR USING THE EMPIRICAL
BAYES METHOD
TUTORIAL**

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<p>16. Abstract</p> <p>The Montana Department of Transportation spends a significant portion of its resources on reconstruction and pavement preservation projects. While the costs for these projects are well quantified, the resulting benefits are not, particularly with respect to traffic safety. In the past, the safety-related benefits attributable to reconstruction and pavement preservation activities have been evaluated by MDT's Safety Management Section using a before/after, case/control analysis for crash trends at various sites around the state. While these methods have been used for many years as a means of predicting safety improvements for roadway segments, recent studies have shown that the Empirical Bayes (EB) method of analysis is more accurate in the estimation of safety improvements for before-after studies. The EB method combines the strengths of before/after, case/control techniques with regression methods for estimating safety-related benefits. The Empirical Bayes method is now used in the USDOT's <i>Interactive, Highway Safety Design Model (IHSDM)</i> and will be used in the <i>SafetyAnalyst</i> software, making this method the standard in professional practice.</p> <p>This tutorial explains the applications of the Empirical Bayes method for both Interstate and non-Interstate roadways. Companion products include a report that describes the theory, application and limitations of the Empirical Bayes method as applied to several Interstate and non-Interstate reconstruction and pavement preservation projects completed in 1997-1998 in the State of Montana, as well as an Excel spreadsheet that facilitates use of this method for MDT's Safety Management Section.</p>		
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1 TUTORIAL GOALS

The goal of this tutorial is to present the Empirical Bayes (EB) method for analyzing before-after crash data in a step-by-step format. The tutorial is designed to be used in conjunction with the companion Empirical Bayes Excel Spreadsheet.

2 EMPIRICAL BAYES METHOD

The Empirical Bayes (EB) process consists of five steps: determining (1) the safety performance function, SPF, (2) the overdispersion parameter, ϕ , (3) the relative weights, α , (4) the estimated expected crashes, π and (5) the index of effectiveness, θ . Each of these steps is described more fully below.

2.1 Determination of the Safety Performance Function, SPF

The first step in the Empirical Bayes process is to determine a unique Safety Performance Function (SPF). The SPF is a mathematical model that predicts an estimate of crash occurrence for a given roadway segment (1). According to Hauer, crash occurrence is best modeled using a multivariate statistical model (2). A model is simply an equation or set of equations that link the expected crash frequency on the roadway to measurable roadway traits such as AADT, length of roadway segment, roadway width, shoulder width, number of lanes, etc.

The SPF is determined from the data collected in the period before any treatments were made to the roadway segment and therefore can consider data available from previously identified “case” and “control” sites to increase the size of the sample and enhance the accuracy of the predictive model. The SPF can then be used to predict the number of crashes expected to occur each year at the “case” sites had there been no improvements to the roadway. Each type of roadway, Interstate and non-Interstate, will have different SPFs to predict the expected number of crashes. The multivariate statistical model used to establish the SPF can be determined using various statistical modeling computer software packages on the market today. *LIMDEP Version 7.0* was used in this investigation to determine the SPFs for the two types of roadways in question. Based on the crash and roadway parameter data available, the SPF was modeled using a multiple linear regression equation that estimates the number of crashes per three years per roadway

segment that occurred in the “before” period. The multiple linear regression equation is of the following form:

$$SPF_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_{p-1} x_{i,p-1} + \varepsilon_i$$

where:

SPF_i denotes the dependant variable (crashes per three years for roadway segment i before any treatment)

x_{i1} through $x_{i,p-1}$ denote the independent, explanatory variables (average annual daily traffic (AADT), number of lanes, lane width, shoulder width, speed limit, etc.) for roadway segment i

β_0 through β_{p-1} denote estimable parameters (determined by *LIMDEP*), β_0 represents the y-intercept value and

ε_i denotes the unexplainable, random error not accounted for in the model.

Using the crash data provided as part of this study, the unique estimated SPFs for Interstate and non-Interstate roadways are described below and detailed in Tables 1 and 2, respectively. In each case, average annual daily traffic (AADT) and segment length were found to be significant in affecting crash occurrence at the 95-percent confidence level (t-statistics ≥ 1.96 confirm significance):

$$SPF_i = \beta_0 + \beta_1(L_i) + \beta_2(AADT_i)$$

where:

L_i denotes the length of roadway segment i

$AADT_i$ denotes the annual average daily traffic per three years on roadway segment i and all other variables are as previously defined.

Variables such as the number of lanes, lane width, shoulder width, speed limit and others were largely invariable across the sample and hence, did not prove to be significant explanatory variables for crash occurrence. For example, lane widths were consistently 12 feet for roadway segments experiencing both high and low crash occurrences precluding any meaningful correlation.

Further, key variables specific to reconstruction and pavement preservation treatments (as compared to direct safety treatments) and their resulting effects on the roadway environment were omitted from the available data set. For example, data describing roadway surface condition (i.e., ride quality, surface friction, degree of rutting or cracking, etc.) prior to and following treatment may have proven significant in predicting the change in crash occurrence for these types of treatments.

Table 1. Model Parameters for Interstate Highways

Variable	$\beta_0, \beta_1, \text{ and } \beta_2$	t-statistic	Standard Error
y-intercept	1.812309	3.584	0.50568
L_i	0.108752	3.435	0.03166
$AADT_i$	0.000167	5.189	0.00003
ϕ	0.078141	2.266	0.03448

Table 2. Model Parameters for Non-Interstate Highways

Variable	$\beta_0, \beta_1, \text{ and } \beta_2$	t-statistic	Standard Error
y-intercept	1.207848	4.302	0.28075
L_i	0.063425	3.506	0.01809
$AADT_i$	0.000560	4.861	0.00012
ϕ	0.182151	2.54	0.07171

With only two significant variables and a small number of roadway segments in the sample, the goodness of fit for either model is low; the adjusted ρ^2 -value for Interstate and non-Interstate roadway segments are 0.240 and 0.201, respectively. A ρ^2 -value equal to 1.0 indicates a perfect model. Also included in Tables 1 and 2 are the estimated overdispersion parameters, ϕ , for each roadway type which are discussed in detail in the next section.

2.2 Determination of the Overdispersion Parameter, ϕ

To estimate a roadway's SPF, it is necessary to assume an underlying probability distribution for the crash frequencies. Historically, crash frequencies were often assumed to follow a Poisson distribution. The Poisson distribution assumes that the mean and variance observed for the crash

frequency variable are equal. Studies have shown that the differences between the crash frequencies and model predictions based on a Poisson distribution are inconsistent, likely resulting from a violation of this equality assumption (3). Therefore, researchers more commonly assume a negative binomial distribution to represent the distribution of crash frequencies (3). One of the parameters used to confirm whether the underlying probability distribution is correctly identified as negative binomial is the overdispersion parameter, ϕ . Data is said to be overdispersed if the variance of the dependent variable exceeds its mean (i.e., violating the constraints of the Poisson distribution). For both Interstate and non-Interstate roadways considered in this investigation, the data was confirmed to be overdispersed (the variance of the crash frequency variable exceeded the mean) as evidenced by a statistically significant ϕ -value at the 95-percent confidence level.

Proceeding with the EB method, this overall overdispersion parameter, representing all roadway segments in combination, is secondly used to account for varying degrees of overdispersion *between* roadway segments attributable to differences in roadway traits and crash occurrences. If each roadway segment were of equal length and had consistent geometric characteristics, traffic characteristics, etc., the overall overdispersion parameter would be directly applicable to each individual roadway segment. However, since the roadway segments vary in length and characteristics, a unique overdispersion parameter, ϕ_i , must be determined for each roadway segment. Segment length is assumed to be a primary determinant affecting individual overdispersion parameter values. Under this assumption, using the overall overdispersion parameter as the overdispersion of each individual segment would skew the model by placing more emphasis on the longer roadway segments (3). To better estimate the expected number of crashes for each individual roadway segment, the overdispersion parameter can be adjusted based on length to represent the individual segment, i :

$$\phi_i = \phi \cdot L_i^\beta$$

where:

ϕ_i denotes the adjusted overdispersion parameter for roadway segment i

ϕ denotes the overall overdispersion parameter for all combined roadway segments

L_i denotes the length of roadway segment i

β is a constant between 0 and 1 (3).

The β -value takes into account the differences in geometric characteristics, traffic characteristics, etc. between the individual roadway segments; if each roadway segment was completely dissimilar from other roadway segments (i.e., had no characteristic similarities to the other roadway segments), $\beta = 0$ and the roadway segment in question would be represented by the overall overdispersion parameter. Alternately, if each of the roadway segments had exactly the same characteristics as all other segments of the roadway, $\beta = 1$ and the overdispersion for the roadway segment in question would be represented by the overall overdispersion parameter adjusted only by the segment length. A β -value somewhere between zero and one is most representative for segments defined along a continuous roadway (3). However, a β -value equal to 1 was assumed for this study to provide the most conservative estimates of future crash occurrences.

An alternative method for determining the adjusted overdispersion parameter assumes a unique gamma distribution for each roadway segment, i (3). The individual overdispersion parameters using this alternate method can be calculated as follows:

$$\phi_i = \phi \cdot SPF_i^\gamma$$

where γ is a constant between 0 and 1 and all other variables are as previously defined(3).

If the parameter γ is set to zero, then the standard negative binomial model is obtained. If γ is greater than zero, then the variance of the gamma distribution decreases as $SPFi$ increases (3). This method of analysis has been used by a number of researchers to determine the individual overdispersion parameter for before-after studies and likely yields more accurate results, however, determination of the γ -value requires analysis not typically employed in practice. Miaou and Lum modeled crash occurrence on rural interstate highways using $\gamma = 1$ (4). Previously published γ -values for similar roadway types may be transferable. Hence, this investigation also assumed $\gamma = 1$.

With no superior method for determining individual overdispersion parameters emerging, this investigation used both methods and carried the two sets of results forward throughout the remainder of the EB process. Tables 3, 4 and 5 and summarize the results.

Table 3. Overdispersion Parameters and Relative Weights for Interstate Roadway Segments

PROJECT I.D.	PROJECT NAME	$\phi_i = \phi L_i$		$\phi_i = \phi SPF_i$	
		Overdispersion Parameter	Relative Weight	Overdispersion Parameter	Relative Weight
I-15-2-(70)116	BUXTON INTERCHANGE. - N & S	0.38	0.114	0.23	0.072
I-15-4(75)200	LINCOLN ROAD - SIEBEN	1.25	0.233	0.32	0.072
I-15-6(28)323	BRADY - NO. & SO. (NORTHBOUND)	0.92	0.202	0.28	0.072
I-90-1(119)74	ALBERTON - EAST & WEST	0.83	0.167	0.32	0.072
I-90-5(53)240	PIPESTONE EAST & WEST	0.66	0.147	0.30	0.072
I-90-7(70)341	MISSION INTERCHANGE - EAST	0.88	0.172	0.33	0.072
I-90-9(81)503	DUNMORE - SOUTH	0.42	0.113	0.26	0.072
I-90-8(131)450	27TH ST. - LOCKWOOD	0.28	0.050	0.41	0.072
I-94-3(50)115	5.3 KM WEST OF HATHAWAY - EAST	1.04	0.215	0.30	0.072
I-94-4(56)129	MILES CITY - EAST & WEST	0.99	0.205	0.30	0.072
I-94-4(57)143	BAKER INTERCHANGE - EAST	0.39	0.119	0.23	0.072
I-94-5(27)163	PRAIRIE COUNTY LINE - EAST	0.50	0.143	0.23	0.072
I-94-6(45)191	DAWSON COUNTY LINE - EAST	1.48	0.252	0.34	0.072

Table 4. Overdispersion Parameters and Relative Weights for Highway Reconstruction Roadway Segments

PROJECT I.D.	PROJECT NAME	$\phi_i = \phi L_i$		$\phi_i = \phi SPF_i$	
		Overdispersion Parameter	Relative Weight	Overdispersion Parameter	Relative Weight
STPP13-1(22)0	REYNOLDS PASS - NORTH	1.57	0.355	0.52	0.154
STPP14-2(12)33	WHITE SULPHUR SPRINGS - SOUTH	1.63	0.357	0.54	0.154
NH16-1(35)23	YELLOWSTONE CO. LINE (N. - S.)	1.16	0.286	0.53	0.154
STPP52-2(20)40	CRESTON NORTH	1.41	0.273	0.68	0.154
NH1-1(37)69	HAPPY'S INN E & W	2.11	0.391	0.60	0.154
STPP13-1(19)65	NORRIS - HARRISON	1.73	0.366	0.55	0.154
NH53-1(18)16	ACTON - NORTHWEST	2.09	0.384	0.61	0.154

Table 5. Overdispersion Parameters and Relative Weights for Highway Preservation Roadway Segments

PROJECT I.D.	PROJECT NAME	$\phi_i = \phi L_i$		$\phi_i = \phi SPF_i$	
		Overdispersion Parameter	Relative Weight	Overdispersion Parameter	Relative Weight
NH1-8(20)72	MALTA - SACO	4.99	0.576	0.67	0.154
STPN5-2(81)79	ELMO - NORTH	2.48	0.385	0.72	0.154
NH11-1(30)14	YANKEE JIM CANYON - NORTH	1.93	0.405	0.52	0.154
NH11-1(31)24	EMIGRANT NORTH - SOUTH	1.81	0.379	0.54	0.154
STPP13-1(27)24	McATEE (NORTH -SOUTH)	2.12	0.445	0.48	0.154
STPN24-1(48)32	CLEARWATER JCT. - EAST	4.35	0.529	0.71	0.154
NH37-2(19)62	ASHLAND - EAST	2.59	0.500	0.47	0.154

2.3 Determination of the Relative Weight, α

To adjust for varying degrees of overdispersion, a relative weight, α_i , is applied to each roadway segment. The segment-specific relative weight is determined as follows:

$$\alpha_i = \frac{1}{1 + SPF_i / \phi_i}$$

where α_i denotes the relative weight applied to roadway segment i and all other variables are as previously defined (1). The roadway segment relative weights for this investigation are provided in Tables 3 through 5.

2.4 Determination of Estimated Expected Crashes, π

Once the previous steps have been completed, the estimate of the expected crashes for a given roadway segment can be calculated using the following equation (1):

$$\pi_i = (\alpha_i) \cdot (SPF_i) + (1 - \alpha_i) (\lambda_i)$$

where:

π_i denotes the expected number of crashes per three years on roadway segment, i

λ_i denotes the actual number of crashes per three years on roadway segment, i

and all other variables are as previously defined.

2.5 Determination of the Index of Effectiveness, θ

The last step in the EB process is to express the resulting effectiveness of any treatment (i.e., roadway reconstruction and pavement preservation improvements, safety improvements, etc.) as a relative difference in crash occurrence between actual and expected. With the expected crash occurrence determined in the previous step and the actual crash occurrence observed, the difference can be calculated directly. However, this direct calculation method does not account for the uncertainty resulting from (1) sampling such a small number of projects to represent the larger population, (2) the resulting low explanatory power (i.e., goodness of fit) of the SPF, (3) the assumptions supporting the determination of the overdispersion parameters and relative weights and (4) the overall underlying data variability project to project. Instead, an index of

effectiveness, θ_i , that takes into account this uncertainty through the data variance observed for each roadway segment must be determined. The variance, σ_i^2 , can be calculated as follows:

$$\sigma_i^2 = (1 - \alpha_i) \pi_i$$

where α_i and π_i are as previously defined.

The variance of the data can also be calculated using the following equation:

$$\sigma_i^2 = SPF_i \left[1 + \frac{SPF_i}{\phi_i \cdot L_i} \right]$$

where all variables are as previously defined.

The index of effectiveness is a function of the previous parameters given by (8):

$$\theta_i = \frac{\lambda_i / \pi_i}{1 + (\sigma_i^2 / \pi_i^2)}$$

where θ_i denotes the index of effectiveness and all other variables are as previously defined.

Finally, the relative difference in crash occurrence between actual and expected conditions is determined as (5):

$$\text{relative difference in crash occurrence} = 100(1 - \theta_i)$$

where all variables are as previously defined and results are expressed as a percentage.

3 USING THE EB EXCEL SPREADSHEET

An Excel spreadsheet is provided to facilitate application of the Empirical Bayes process, previously outlined, for: (1) Interstate, (2) two-lane highway preservation and (3) two-lane highway restoration projects. The SPF and overall overdispersion parameter have already been established for each roadway type as part of this larger study.

The first component of the spreadsheet provides an overview of the EB process as shown in Figure 1. The second component of the spreadsheet consists of two tables that determine the effectiveness of roadway treatments (based on the two alternate overdispersion parameter, ϕ_i , calculations) using the process outlined in Section 2. The user is required to input values into the yellow highlighted cells in the worksheet shown in Figure 2. Once the information is included in

the spreadsheet, the percent relative difference between actual and expected crashes will automatically be determined. The information required for the spreadsheet includes:

- Project ID
- Project Name
- Milepost at the beginning of the project segment (MP Begin) and the Milepost at the end of the project segment (MP End)
- AADT on the segment
- Number of actual crashes on the segment

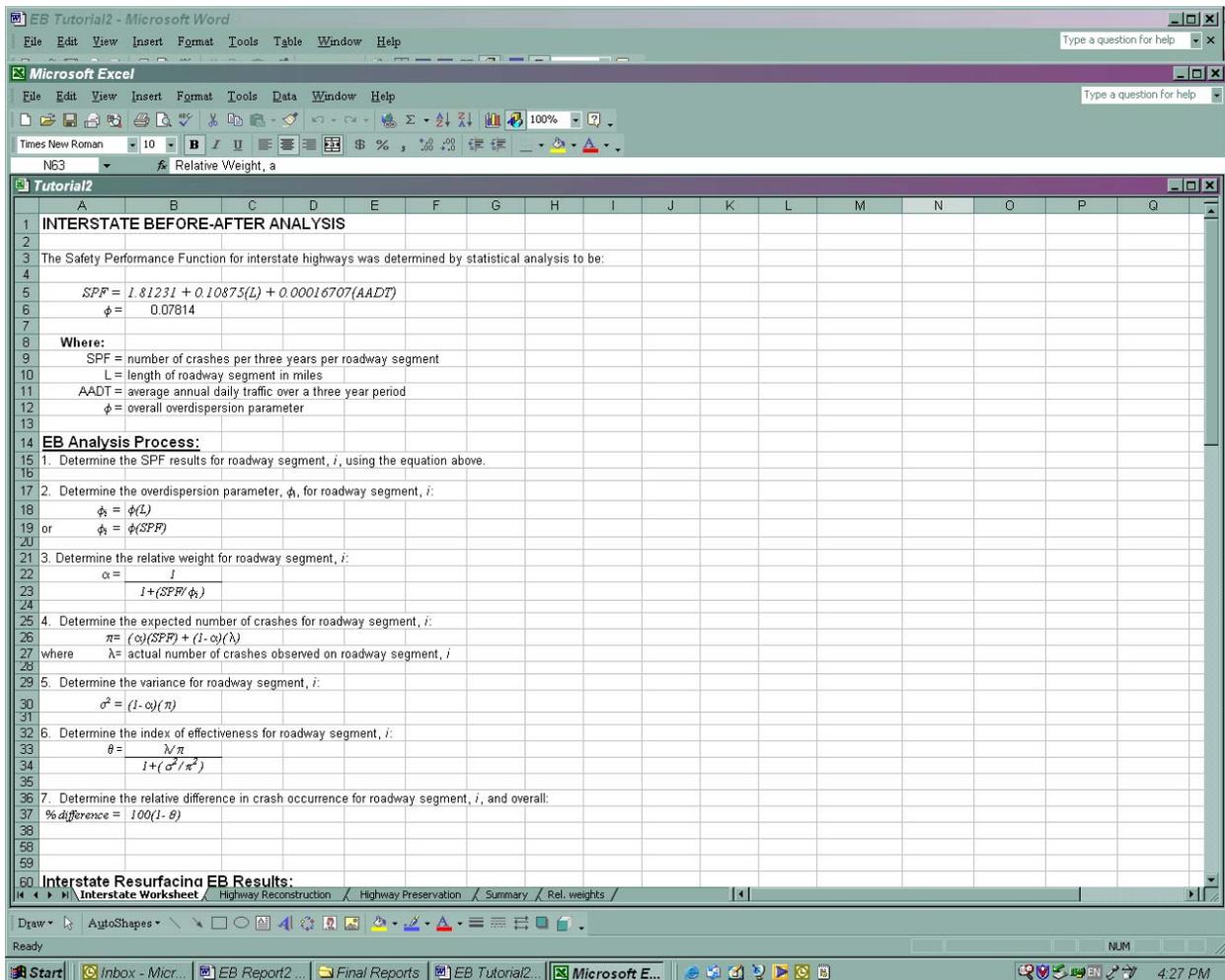


Figure 1. Overview of the EB Method

results for all roadway reconstruction projects in combination are highlighted in green in the spreadsheet and are located at the bottom of the % Lower than Expected and the % Higher than Expected columns (see Figure 3).

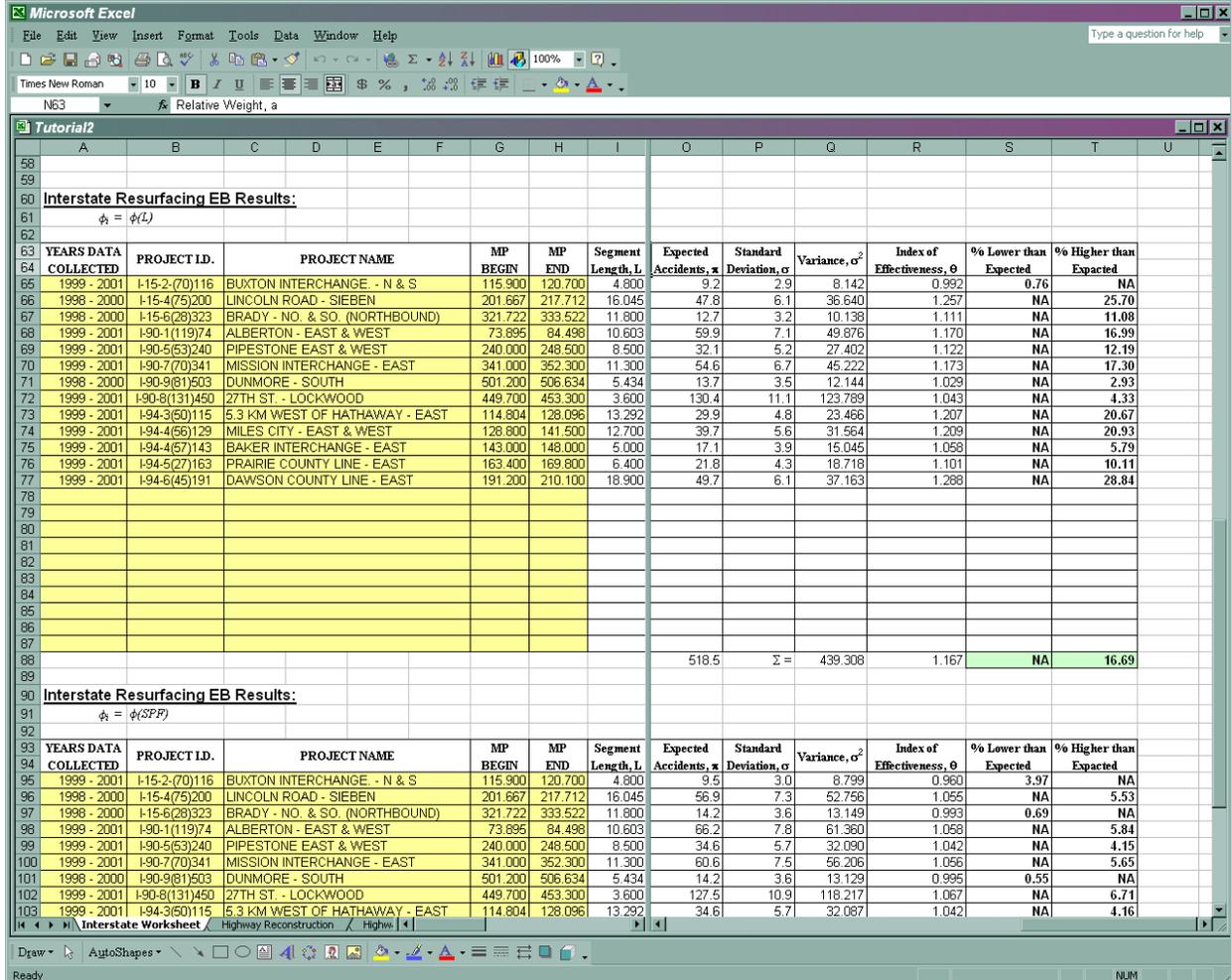


Figure 3. Relative Difference Display

5 LIMITATIONS OF THE EB METHOD SPREADSHEET

The primary limitation of the EB method as applied in this spreadsheet is that the SPF was estimated using an aggregate three years of crash data (i.e., crash frequencies per three years per roadway segment). Hence, to accurately apply this SPF model, the units of crash frequencies per *three* years per roadway segment need to be maintained (i.e., annual crash data cannot be used in place of three-year aggregated data without re-estimating the SPF model).

A more minor limitation pertaining to the use of the spreadsheet occurs when new rows of information are added into Excel. All of the calculations for individual roadway segments will remain unaltered, however the final percent relative difference in crashes by roadway type could be miscalculated if the user does not modify the cell formula to include the additional rows. The user should double-check that the summations of the *Actual Crashes*, the *Expected Crashes*, and the *Variance* columns are all summed correctly, so that the final results are accurate.

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