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*MONTANA DEPARTMENT OF  
TRANSPORTATION*

**ROAD DESIGN MANUAL**

**CHAPTER TEN  
VERTICAL ALIGNMENT**



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## Chapter Ten

# VERTICAL ALIGNMENT

The highway vertical alignment plays a significant role in a highway's safety, aesthetics and project costs. Chapter Twelve "Geometric Design Tables" provides numerical criteria for various vertical alignment elements. Chapter Ten provides additional guidance on these and other vertical alignment elements, including laying out a profile grade line, maximum and minimum allowable grades, critical lengths of grade, climbing lanes, vertical curvature computations and vertical clearances.

### 10.1 DEFINITIONS/NOMENCLATURE

1. Bus. A heavy vehicle involved in the transport of passengers on a for-hire, charter or franchised transit basis.
2. Critical Length of Grade. The maximum length of a specific upgrade on which a loaded truck can operate without experiencing a specified reduction in speed.
3. Gradient. The rate of slope between two adjacent vertical points of intersection (VPI) expressed as a percent. The numerical value for percent of grade is the vertical rise or fall in feet (meters) for each 100' (100 m) of horizontal distance. Upgrades in the direction of stationing are identified as plus (+). Downgrades are identified as minus (-).
4. Heavy Vehicles. Any vehicle with more than four wheels touching the pavement during normal operation. Heavy vehicles collectively include trucks, recreational vehicles and buses.
5. K-Values. The horizontal distance needed to produce a 1% change in gradient.
6. Level Terrain. Level terrain is generally considered to be flat, and has minimal impact on vehicular performance. Highway sight distances are either long or could be made long without major construction expense.
7. Momentum Grade. A site where an upgrade is preceded by a downgrade, thereby allowing a truck to increase its speed on the upgrade. This increase in speeds allows the designer to use a higher speed reduction in the critical length of grade figure.

8. Mountainous Terrain. Longitudinal and transverse changes in elevation are abrupt, and benching or side hill excavation are frequently required to provide the desirable highway alignment. Mountainous terrain aggravates the performance of trucks relative to passenger cars, resulting in some trucks operating at crawl speeds.
9. Performance Curves. A set of curves which illustrate the effect grades will have on the design vehicle's acceleration and/or deceleration.
10. Profile Grade Line. A series of tangent lines connected by vertical curves. It is typically placed along the roadway centerline of undivided facilities and at the edges of the two roadways on the median side on divided facilities.
11. Recreational Vehicle. A heavy vehicle, generally operated by a private motorist, engaged in the transportation of recreational equipment or facilities; examples include campers, boat trailers, motorcycle trailers, etc.
12. Rolling Terrain. The natural slopes consistently rise above and fall below the roadway grade and, occasionally, steep slopes present some restriction to the desirable highway alignment. In general, rolling terrain generates steeper grades, causing trucks to reduce speeds below those of passenger cars.
13. Spline Curve. A curve, drawn using a flexible template to meet field conditions.
14. Truck. A heavy vehicle engaged primarily in the transport of goods and materials, or in the delivery of services other than public transportation. For geometric design and capacity analyses, trucks are defined as vehicles with six or more tires. Data on trucks are compiled and reported by the Transportation Planning Division.
15. VPC (Vertical Point of Curvature). The point at which a tangent grade ends and the vertical curve begins.
16. VPI (Vertical Point of Intersection). The point where the extension of two tangent grades intersect.
17. VPT (Vertical Point of Tangency). The point at which the vertical curve ends and the tangent grade begins.

## 10.2 DESIGN PRINCIPLES AND PROCEDURES

### 10.2.1 General Controls for Vertical Alignment

As discussed elsewhere in Chapter Ten, the design of vertical alignment involves, to a large extent, complying with specific limiting criteria. These include maximum and minimum grades, sight distance at vertical curves and vertical clearances. In addition, the designer should adhere to certain general design principles and controls which will determine the overall safety of the facility and will enhance the aesthetic appearance of the highway. These design principles for vertical alignment include:

1. Consistency. Use a smooth grade line with gradual changes, consistent with the type of highway and character of terrain, rather than a line with numerous breaks and short lengths of tangent grades.
2. Environmental Impacts. Vertical alignment should be properly coordinated with environmental impacts (e.g., encroachment onto wetlands). The Engineering Bureau within the Environmental Services Office is responsible for evaluating environmental impacts.
3. Long Grades. On a long ascending grade, it is preferable to place the steepest grade at the bottom and flatten the grade near the top.
4. Intersections. Maintain moderate grades through intersections to facilitate turning movements. See Chapter Thirteen for specific information on vertical alignment through intersections.
5. Roller Coaster. Avoid using a "roller-coaster" type of profile. They may be proposed in the interest of economy, but they are aesthetically undesirable and may be hazardous.
6. Broken-Back Curvature. Avoid "broken-back" grade lines (two crest or sag vertical curves separated by a 500' (150 m) or less tangent section). One long vertical curve is more desirable.
7. Coordination with Natural/Man-Made Features. The vertical alignment should be properly coordinated with the natural topography, available right-of-way, utilities, roadside development and natural/man-made drainage patterns.
8. VPI Locations. Set VPI locations at even 50' (10 m) stations if practical.

### **10.2.2 Coordination of Horizontal and Vertical Alignment**

Horizontal and vertical alignment should not be designed separately, especially for projects on new alignment. Their importance demands that the designer carefully evaluate the interdependence of these two highway design features. This will enhance highway safety and improve the facility's operation. The following should be considered in the coordination of horizontal and vertical alignment:

1. Balance. Curvature and grades should be in proper balance. Maximum horizontal curvature with flat grades or flat curvature with maximum grades does not achieve this desired balance. A compromise between the two extremes produces the best design relative to safety, capacity, ease and uniformity of operations and a pleasing appearance.
2. Coordination. Vertical curvature superimposed upon horizontal curvature (i.e., vertical and horizontal P.I.'s at approximately the same stations) generally results in a more pleasing appearance and reduces the number of sight distance restrictions. Successive changes in profile not in combination with the horizontal curvature may result in a series of humps visible to the driver for some distance, which may produce an unattractive design. However, under some circumstances, superimposing the horizontal and vertical alignment must be tempered somewhat by Comment #'s 3 and 4 as follows.
3. Crest Vertical Curves. Sharp horizontal curvature should not be introduced at or near the top of pronounced crest vertical curves. This is undesirable because the driver cannot perceive the horizontal change in alignment, especially at night when headlight beams project straight ahead into space. This problem can be avoided if the horizontal curvature leads the vertical curvature or by using design values which exceed the desirable.
4. Sag Vertical Curves. Sharp horizontal curves should not be introduced at or near the low point of pronounced sag vertical curves or at the bottom of steep vertical grades. Because visibility to the road ahead is foreshortened, only flat horizontal curvature will avoid an undesirable, distorted appearance. At the bottom of long grades, vehicular speeds often are higher, particularly for trucks, and erratic operations may occur, especially at night.
5. Passing Sight Distance. In some cases, the need for frequent passing opportunities and a higher percentage of passing sight distance may supersede the desirability of combining horizontal and vertical alignment. In these cases, it may be necessary to provide long tangent sections to secure sufficient passing sight distance.

6. Intersections. At intersections, horizontal and vertical alignment should be as flat as practical to provide designs which produce sufficient sight distance and gradients for vehicles to slow or stop. See Chapter Thirteen.
7. Divided Highways. On divided facilities with wide medians, it is frequently advantageous to provide independent alignments for the two 1-way roadways. Where traffic justifies a divided facility, a superior design with minimal additional cost generally can result from the use of independent alignments.
8. Residential Areas. Design the alignment to minimize nuisance factors to neighborhoods. Minor adjustment to the horizontal or vertical alignment may increase the buffer zone between the highway and residential areas.
9. Aesthetics. Design the alignment to enhance attractive scenic views of rivers, rock formations, parks, golf courses, etc. The highway should head into rather than away from those views that are considered to be aesthetically pleasing. The highway should fall towards those features of interest at a low elevation and rise toward those features which are best seen from below or in silhouette against the sky.

### **10.2.3 Profile Grade Line**

#### **10.2.3.1 General**

The profile grade line is perhaps the roadway geometric characteristic which has the greatest impact on a facility's costs, aesthetics, safety and operation. The profile grade is a series of tangent lines connected by parabolic vertical curves.

The designer must carefully evaluate many factors when establishing the profile grade line. These include:

1. maximum and minimum gradients;
2. sight distance criteria;
3. earthwork balance;
4. adjacent land use and values;
5. coordination with other geometric features (e.g., cross section);
6. highway safety;
7. topography/terrain;
8. right-of-way;
9. utilities;
10. truck performance;

11. highway intersections and interchanges;
12. railroad/highway crossings;
13. bridges and drainage structures;
14. high water levels;
15. drainage considerations;
16. water table elevations;
17. snow drifting;
18. types of soil;
19. urban/rural location;
20. aesthetics/landscaping;
21. construction costs.;
22. environmental impacts;
23. driver expectations;
24. airport flight paths (e.g., grades and lighting); and
25. pedestrian and handicapped accessibility.

The following sections discuss the establishment of the profile grade line in more detail.

### 10.2.3.2 Profile Grade Line Locations

The location of the profile grade line on the roadway cross section varies according to the highway type, whether or not the facility is divided or undivided, and the median type. The profile grade line locations are shown in the typical cross section figures provided in Section 11.7. The recommended profile grade line and axis of rotation locations for various typical sections are as follows:

1. Freeways With Depressed Median Sections (Medians 36' (11 m) to 76' (23 m) wide). A profile grade line is provided for each roadway, which is established on the outside edge of each median shoulder. See Figures 11.7A, 11.7B, 11.7C and 11.7D. The axes of rotation are about the profile grade lines.
2. Freeways With Depressed Median Sections (Medians 76' (23 m) and Wider). Freeways with wide medians typically use independent alignments for both roadways. Therefore, a profile grade line is necessary for each roadway and is provided at the centerline of each roadway. See Figure 11.7E. For superelevated sections, the profile grade less normal crown is shown at the low side of the superelevation between the travel lane and shoulder on each roadway. See Figure 11.7F. The axes of rotation are about the profile grade less normal crown lines.

3. Flush Median Section (Freeways and Non-Freeways). For divided facilities with flush medians, including those with concrete median barriers, there is typically only one profile grade line at the centerline of the roadway. See Figures 11.7G, 11.7H, 11.7I and 11.7J. The axis of rotation is about the profile grade line.
4. Raised Median Section (Non-Freeways). For raised median sections, there is only one profile grade line at the centerline of the median or roadway. See Figures 11.7K and 11.7L. The axis of rotation is about the profile grade line.
5. Two-Lane Rural Highways. For 2-lane rural highways, there is typically only one profile grade line at the centerline of the roadway. See Figure 11.7M. For superelevated sections, the profile grade line less normal crown is shown at the low side between the travel lane and shoulder. See Figure 11.7N. The axis of rotation is about the profile grade less normal crown line.
6. Curbed Urban Undivided Facilities. The profile grade line will be placed at the centerline of the travelway. In addition, separate profiles at the top back of curb should be provided where necessary to ensure positive drainage and to match existing development. See Figures 11.7O and 11.7P. The axis of rotation is about the profile grade.

### 10.2.3.3 Urban Grade Design

Laying out profile grade lines in urban areas often is more complicated due to limited right-of-way, closely spaced intersections, the need to meet existing roadside development and accommodating drainage on curbed streets. The following provides several considerations that should be reviewed when developing a profile grade line on an urban project:

1. Vertical Curves. Long vertical curves on urban streets are generally impractical. The designer will typically need to lay out the profile grade line to meet existing conditions. Therefore, no minimum vertical curve lengths are provided for urban streets. Where practical, VPI's should be located at or near the centerlines of cross streets. Vertical curves will not be required when the algebraic difference in grades is less than 1.0%. However, the use of vertical curves should be evaluated when the algebraic difference in grades is greater than 0.5%. In addition, at signalized and stopped controlled intersections, some flattening of the approaches may be required, See Chapter Twenty-eight in the *Traffic Engineering Manual*.

2. Surface Drainage. Urban streets will often have curbs, which may complicate the layout of the profile grade to facilitate drainage. Special care should be taken to avoid flat spots where water may pond. Section 10.3.2 provides the minimum gradients for curbed streets. It will be necessary to develop separate profiles at the top of curb to promote positive drainage. Design curb elevations to permit drainage flow into the gutter and to avoid ponding of water behind the curb. At intersections, the surface drainage should preferably be intercepted upstream of the intersection. Where surface drainage is provided across intersecting streets, the drainage depression should have a minimum radius of 12' (3.6 m).
3. Spline Curves. Spline curves can be helpful in laying grades in urban areas where it is necessary to meet numerous elevation restrictions in relatively short distances. Spline curves may also be used when developing profiles for the top of curbs. The designer will need to tie these curves to the profile grade line at the beginning and end. Show elevations along spline curves at 10' (5 m) intervals.
4. Existing Roadside Development. Where roadside development is extensive and the general elevation on one side is higher than on the other, an asymmetrical section may be required. The crown point may be offset from the centerline and the total drop from crown line to gutter line will be more than normal on one side and less than normal on the other. Asymmetrical features must be clearly defined in the typical sections and shown in the cross sections. It will also be necessary to provide separate profiles at the top of curbs to match existing development. Providing an asymmetrical cross section may be preferred to reshaping existing sidewalks, parking lots, lawns, etc., to meet the revised profile.
5. Earthwork Balance. In general, balancing of earthwork is typically impractical in urban areas. An excess of excavation is preferable to the need for borrow, due to the generally higher cost of borrow in urban areas.
6. Limited Right-of-Way. Careful consideration must be given when substantially lowering or raising the profile grade line. This will often result in more right-of-way impacts (e.g., longer fill and cut slopes) where right-of-way is often restricted along urban streets.

#### **10.2.3.4 Earthwork Balance**

Where practical and where consistent with other project objectives, design the profile grade line to provide a balance of earthwork. This should not be achieved, however, at the expense of smooth grade lines and sight distance requirements at vertical curves.

Ultimately, a project-by-project assessment will determine whether a project will require borrow, excess or be balanced. For additional information on balancing earthwork, see Section 5.2.4.

#### **10.2.3.5 Field Recommendations**

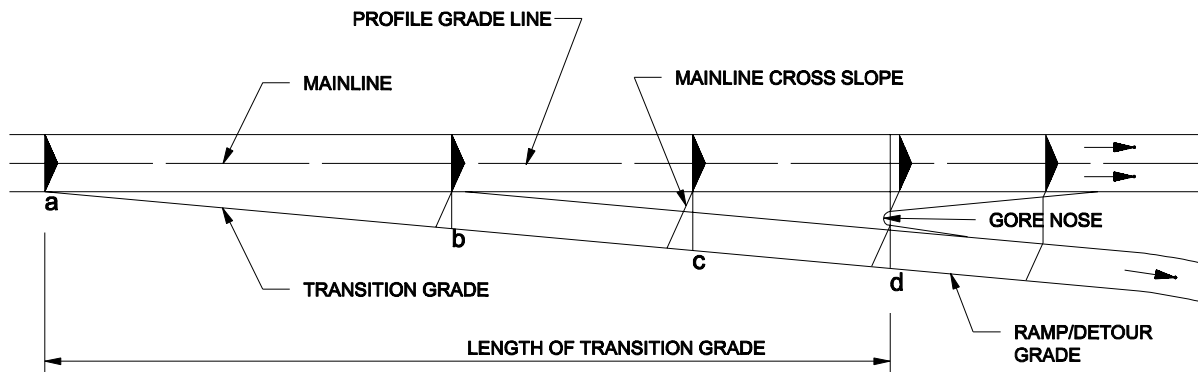
Recommendations should be made at the preliminary field review for addressing special grade controls. For example, recommendations should be provided for the profile grade line across flats, lake beds, sloughs, creek bottoms, important intersecting roads, and in front of improved property, at places subject to snow drifting and at other places requiring special attention. The designer should review and consider these recommendations when establishing the tentative profile grade line.

#### **10.2.3.6 Ties with Existing Highways/Adjoining Projects**

A smooth transition is needed between the proposed profile grade line of the project and the existing grade line of an adjacent highway section or the proposed grade line for an adjacent project. Grade lines should be reviewed for a distance of at least 2000' (600 m) beyond the beginning and end of a project to ensure adequate sight distance. Connections should be made which are compatible with the design speed of the new project and which can be used if the adjoining road section is reconstructed.

#### **10.2.3.7 Transition Grades**

The transition grade (previously called the spline grade) is the grade line where the taper from the mainline begins for freeway ramps, turning roadways, transitions from 2-lane to 4-lane sections, detours, crossovers, etc., to the point where the ramp or roadway grade becomes independent of the mainline (i.e., at the gore nose). Figure 10.2A illustrates the location of the transition grade for ramps and detours. The transition grade is dependent on the mainline until the ramp or roadway becomes independent (i.e., the cross slope of grade lines must meet the transition grade at section "d" shown in Figure 10.2A). Site constraints may limit the ramp length such that the PC of a curve on the ramp must be located at the beginning of the independent ramp grade (d. on Figure 10.2A). For these cases, the cross slope of the ramp may vary from the cross slope of the mainline to permit a transition to the superelevation on the curve.



### TRANSITION GRADE (Ramps/Detours)

**Figure 10.2A**

For ramps, the transition grade should extend from the beginning of the taper from the mainline to a point where the distance between the edge of the travel lane on the mainline to the edge of the travel lane on the ramp is 8 feet (2.4 meters).

For detours, the transition grade should extend from the beginning of the taper from the mainline to a point where the distance between the edge of the travel lane on the mainline to the edge of the travel lane on the detour is 4 feet (1.2 meters). See Section 15.3 for additional information regarding transition grades for detours.

#### 10.2.3.8 Bridges

The design of profile grade lines must be carefully coordinated with any bridges or drainage structures within the project limits. The following will apply:

1. Vertical Clearances. The criteria in Section 10.6 must be met where a new roadway and structure will be constructed over an existing roadway or where the roadway will be reconstructed under an existing bridge. When laying the preliminary grade line, an important element in determining available vertical clearance is the assumed structure depth. This will be based on the structure type, span lengths and depth/span ratio. The road designer should contact the Bridge Bureau to obtain an estimated depth of structure. If no information is available, the designer should assume a 20.0' (6.2 m) to 22.0' (6.5 m) distance

between the finished grade of the roadway and the finished grade of the bridge deck. For final design, the designer must coordinate with the Bridge Bureau to determine the roadway and bridge grade lines.

2. Highway Under Bridge. Where practical, the low point of a roadway sag vertical curve should not be within the shadow of the bridge. This will help minimize ice accumulations, and it will reduce the ponding of water which may weaken the earth foundation beneath the bridge. To achieve these objectives, the low point of a roadway sag should be approximately 100' (30 m) from the bridge.
3. Bridges Over Water. Where the proposed facility will cross bodies of water, the bridge elevation must be consistent with the necessary waterway opening to meet the Department's hydraulic requirements. The road designer provides the Bridge Bureau with the preliminary grade line. The bridge designer determines minimum bridge elevations based on the hydraulic requirements. The designer must coordinate with the Hydraulics Section and Bridge Bureau to determine the final approach roadway elevation to meet the necessary bridge elevation.
4. Railroad Bridges. Any proposed facilities over railroads must meet the applicable criteria (e.g., vertical clearances, structure type and depth). The designer should contact the Bridge Bureau and the Utilities Section for more information.
5. High Embankments. The designer should consider the impacts of high embankments on structures. This will increase the span length thus increasing structure costs.
6. Low Point. It is desirable to locate the low point of a sag vertical curve off the bridge deck.

#### **10.2.3.9 Soils**

The type of earth material encountered often influences the grade line at certain locations. If rock is encountered, for example, it may be more economical to raise the grade and reduce the rock excavation. Soils which are unsatisfactory for embankment or cause a stability problem in cut areas may also be determining factors in establishing a grade line. The designer should coordinate the development of the profile grade with the Materials Bureau, which will be responsible for conducting the soils survey.

### 10.2.3.10 Drainage/Snow

The profile grade line should be compatible with the roadway drainage design and should minimize snow drift problems. Consider the following:

1. Culverts. The roadway elevation should provide at least the minimum cover indicated in the culvert fill height tables in Chapter Seventeen. Desirably, culverts should not extend above the top of the subgrade. Do not locate the low points of sag vertical curves directly over culverts. This is so that if flooding overtops the roadway, the chance of culverts being washed out will be minimized. See Chapter Seventeen for additional information on culvert designs.
2. Coordination with Geometrics. The profile grade line must reflect compatibility between drainage design and roadway geometrics. These include the design of sag and crest vertical curves, spacing of inlets on curbed facilities, impacts on adjacent properties, superelevated curves, intersection design elements and interchange design elements. For example, avoid placing sag vertical curves in cuts and placing long crest vertical curves on curbed pavements.
3. Snow Drifting. Where practical, the profile grade line should be at least 2' (0.5 m) above the natural ground level on the windward side of the highway to prevent snow from drifting onto the roadway and to promote snow blowing off the roadway. See Section 11.4.2 for additional criteria to minimize snow drifting.

### 10.2.3.11 Erosion Control

To minimize erosion, the designer should consider the following relative to the grade line:

1. Minimize the number of deep cuts and high fill sections.
2. Conform to the contour and drainage patterns of the area.
3. Make use of natural land barriers and contours to divert runoff and confine erosion and sedimentation.
4. Minimize the amount of disturbance.
5. Make use of existing vegetation.
6. Reduce slope length and steepness and ensure that erosion is confined to the right-of-way and does not deposit sediment on or erode away adjacent land.

7. Avoid locations having high base erosion potential.
8. Avoid cut or fill sections in seepage areas.

#### **10.2.3.12 Project Types**

In addition to reconstruction projects that involve significant grade modifications, a new profile grade should be shown for pavement pulverization projects and projects that involve major surfacing rehabilitation.

It is not necessary to provide profile grades for overlays or other minor surfacing rehabilitation projects. If the need for a profile grade is in question, consult with Construction personnel.



## **10.3 GRADES**

### **10.3.1 Maximum Grades**

Chapter Twelve presents the Department's criteria for maximum grades based on functional classification, urban/rural location, type of terrain, and in some cases, design speed. The maximum grades should be used only where absolutely necessary. Where practical, use grades flatter than the maximum.

### **10.3.2 Minimum Grades**

The following provides the Department's criteria for minimum grades:

1. Uncurbed Roads. Desirably, a 0.5% minimum longitudinal grade should be provided. Level longitudinal gradients may be acceptable on pavements in fills and which are adequately crowned to drain laterally. In cuts, minimum longitudinal gradients of 0.2% are acceptable.
2. Curbed Streets. The centerline profile on highways and streets with curbs should desirably have a minimum longitudinal gradient of 0.5%. Longitudinal gradients of at least 0.4% are acceptable. Because surface drainage is retained within the roadway, the longitudinal gradients must be steeper on curbed streets than uncurbed roadways to avoid ponding of water on the roadway surface.

### **10.3.3 Critical Length of Grade**

Critical length of grade is the maximum length of a specific upgrade on which a loaded truck can operate without experiencing a specified reduction in speed. The highway gradient in combination with the length of grade will determine the truck speed reduction on upgrades. Refer to the *Traffic Engineering Manual* to determine the critical length of grade



#### **10.4 TRUCK-CLIMBING LANES**

The Traffic Engineering Section will typically determine the need for truck-climbing lanes and will provide the design details for these lanes where they are warranted. For information on truck-climbing lanes, see Chapters Twenty-six and Thirty in the *Traffic Engineering Manual*.



## 10.5 VERTICAL CURVES

### 10.5.1 Crest Vertical Curves

Crest vertical curves are in the shape of a parabola. The basic equations for determining the minimum length of a crest vertical curve are:

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} \quad (\text{Equation 10.5-1})$$

$$L = KA \quad (\text{Equation 10.5-2})$$

Where:

L	=	length of vertical curve, ft (m)
A	=	algebraic difference between the two tangent grades, %
S	=	sight distance, ft (m)
$h_1$	=	height of eye above road surface, ft (m)
$h_2$	=	height of object above road surface, ft (m)
K	=	horizontal distance needed to produce a 1% change in gradient

The length of the crest vertical curve will depend upon "A" for the specific curve and upon the selected sight distance, height of eye and height of object. The following discusses the selection of these values. For design purposes, the calculated length of curve based on the rounded K-value should be rounded up to the next highest 50' (20 m) increment.

The principal control in the design of crest vertical curves is to ensure that, at a minimum, stopping sight distance (SSD) is available throughout the curve. Figure 10.5A presents the K-values for stopping sight distance. The following discusses the application of the K-values:

1. Passenger Cars (Level Grade). Figure 10.5A presents minimum K-values for passenger cars. These are calculated by assuming  $h_1 = 3.5'$  (1.080 m),  $h_2 = 2'$  (0.60 m) and  $S = \text{SSD}_{\text{minimum}}$  in the basic equation for crest vertical curves (Equation 10.5-1). The minimum values represent the lowest acceptable sight distance on a facility. However, the designer should provide a design in which the K-values meet the greatest practical SSD.

## US Customary

DESIGN SPEED (mph)	ROUNDED SSD FOR DESIGN <sup>1</sup> (ft)	CALCULATED K-VALUES <sup>2</sup>	K-VALUES ROUNDED FOR DESIGN <sup>2</sup>
	Minimum	2' Object $K = \frac{S^2}{2158}$	Desirable
20	115	6.1	7
25	155	11.1	12
30	200	18.5	19
35	250	29.0	29
40	305	43.1	44
45	360	60.1	61
50	425	83.7	84
55	495	113.5	114
60	570	150.6	151
65	645	192.8	193
70	730	246.9	247
75	820	311.6	312

## Metric

DESIGN SPEED (km/h)	ROUNDED SSD FOR DESIGN <sup>1</sup> (m)	CALCULATED K-VALUES <sup>2</sup> ( $K=S^2/658$ )	K-VALUES ROUNDED FOR DESIGN <sup>2</sup>
		600 mm Object	
30	35	1.9	2
40	50	3.8	4
50	65	6.4	7
60	85	11.0	11
70	105	16.8	17
80	130	25.7	26
90	160	38.9	39
100	185	52.0	52
110	220	73.6	74
120	250	95.0	95

Notes:

1. Stopping sight distances (SSD) are from Figure 8.6A.
2. K-values are calculated using rounded stopping sight distances, eye height of 3.5' (1.080 m) and object height of 2' (0.60 m). AASHTO K-values are based on the calculated stopping sight distance.

**K-VALUES FOR CREST VERTICAL CURVES  
(Level Grades)**

**Figure 10.5A**

2. Minimum Length. For aesthetics, the suggested minimum length of a crest vertical curve on a rural highway is 1000' (300 m). Where the algebraic difference in grades (A) is small, the calculated curve lengths may actually be zero. However, angle points are not allowed on rural highways. Therefore, the minimum length of curve is based on Equation 10.5-3.

US Customary

Metric

$$L_{\min} = 3V$$

$$L_{\min} = 0.6 V$$

(Equation 10.5-3)

Where:

$$\begin{array}{ll} L_{\min} & = \text{minimum length of vertical curve, ft (m)} \\ V & = \text{design speed, mph (km/h)} \end{array}$$

3. Drainage. Drainage should be considered in the design of crest vertical curves where curbed sections are used. Drainage problems should not be experienced if the vertical curvature is sharp enough so that a minimum longitudinal grade of at least 0.3% is reached at a point about 50' (15 m) from either side of the apex. To ensure that this objective is achieved, the length of the vertical curve should be based upon a K-value of 167 (50) or less. For crest vertical curves on curbed sections where this K-value is exceeded, the drainage design should be more carefully evaluated near the apex.

For uncurbed sections of highway, drainage should not be a problem at crest vertical curves. However, the adjacent roadside ditch should have minimum longitudinal gradient of 0.2% in the vicinity of the vertical curve. Grading the ditches separately from the mainline may be necessary to achieve the desired grade.

4. Passing Sight Distance. At some locations, it may be desirable to provide passing sight distance in the design of crest vertical curves. On rural reconstruction projects, the designer should attempt to provide passing sight distance over as much of the highway length as practical. It will generally not be cost effective, however, to make significant improvements to the horizontal and vertical alignment solely to increase the available passing sight distance. Section 8.6.2 discusses the application and design values for passing sight distance. These "S" values are used in the basic equations for crest vertical curves (Equations 10.5-1 and 10.5-2). The height of eye ( $h_1$ ) is 3.5' (1.080 m) and the height of object ( $h_2$ ) is 3.5' (1.080 m). Figure 10.5B presents the minimum K-values for determining passing sight distance.

## US Customary

DESIGN SPEED (mph)	MINIMUM PASSING SIGHT DISTANCE FOR DESIGN <sup>(1)</sup> (ft)	CALCULATED K-VALUES <sup>(2)</sup> ( $K=S^2/2800$ )	K-VALUES ROUNDED FOR DESIGN <sup>(2)</sup>
30	1090	424.3	424
35	1280	585.1	585
45	1625	943.1	943
50	1835	1202.6	1203
55	1985	1407.2	1407
60	2135	1627.9	1628
70	2480	2196.6	2197
75	2580	2377.3	2377

## Metric

DESIGN SPEED (km/h)	MINIMUM PASSING SIGHT DISTANCE FOR DESIGN <sup>(1)</sup> (m)	CALCULATED K-VALUES <sup>(2)</sup> ( $K=S^2/864$ )	K-VALUES ROUNDED FOR DESIGN <sup>(2)</sup>
50	345	137.8	138
60	410	194.6	195
70	485	272.3	272
80	540	337.5	338
90	615	437.8	438
100	670	519.6	520
110	730	616.8	617
120	775	695.2	695

Notes:

1. *Passing sight distances are from Figure 8.6C.*
2. *K-values are calculated using the passing sight distance, eye height of 3.5' (1.080 m) and object height of 3.5' (1.080 m).*

**K-VALUES FOR PASSING SIGHT DISTANCE  
(Crest Vertical Curves)**

**Figure 10.5B**

### 10.5.2 Sag Vertical Curves

Sag vertical curves are in the shape of a parabola. Typically, they are designed to allow the vehicular headlights to illuminate the roadway surface (i.e., the height of object = 0.0' (m) for a given distance "S." These assumptions yield the following basic equations for determining the minimum length of sag vertical curves:

$$L = \frac{AS^2}{200h_3 + 3.5S} \quad (\text{Equation 10.5-4})$$

$$L = KA \quad (\text{Equation 10.5-5})$$

Where:

L	=	length of vertical curve, ft (m)
A	=	algebraic difference between the two tangent grades, %
S	=	sight distance, ft (m)
$h_3$	=	height of headlights above pavement surface, ft (m)
K	=	horizontal distance needed to produce a 1% change in gradient

The length of the sag vertical curve will depend upon "A" for the specific curve and upon the selected sight distance and headlight height. For design purposes, the calculated length of curve based on the rounded K-value should be rounded up to the next highest 50' (20 m) increment.

The principal control in the design of sag vertical curves is to ensure that, at a minimum, stopping sight distance (SSD) is available for headlight illumination throughout the curve. The design assumes that there is a 1° upward divergence of the light beam from the longitudinal axis of the headlights. Figure 10.5C presents the K-values for stopping sight distances. The following discusses the application of the K-values:

1. Passenger Cars. Figure 10.5C presents minimum and desirable K-values for passenger cars. These are calculated by assuming  $h_3 = 2'$  (0.600 m) and  $S = \text{SSD}$  in the basic equation for sag vertical curves (Equation 10.5-4). The minimum values represent the lowest acceptable sight distance on a facility.

## US Customary

DESIGN SPEED (mph)	ROUNDED SSD FOR DESIGN <sup>1</sup> (ft)	CALCULATED K-VALUES <sup>2</sup> ( $K=S^2/(400+3.5(S))$ )	K-VALUES ROUNDED FOR DESIGN <sup>2</sup>
20	115	16.5	17
25	155	25.5	26
30	200	36.4	37
35	250	49.0	49
40	305	63.4	64
45	360	78.1	79
50	425	95.7	96
55	495	114.9	115
60	570	135.7	136
65	645	156.5	157
70	730	180.3	181
75	820	205.6	206

## Metric

DESIGN SPEED (km/h)	ROUNDED SSD FOR DESIGN <sup>1</sup> (m)	CALCULATED K-VALUES <sup>2</sup> ( $K=S^2/(120+3.5(S))$ )	K-VALUES ROUNDED FOR DESIGN <sup>2</sup>
30	35	5.1	6
40	50	8.5	9
50	65	12.2	13
60	85	17.3	18
70	105	22.6	23
80	130	29.4	30
90	160	37.6	38
100	185	44.6	45
110	220	62.8	55
120	250	72.7	63

## Notes:

1. Stopping sight distances (SSD) are from Figure 8.6A.
2. K-values calculated using rounded stopping sight distances and a headlight height ( $h_3$ ) of 2' (0.600 m). AASHTO K-values are based on the calculated stopping sight distance.

## K-VALUES FOR SAG VERTICAL CURVES

Figure 10.5C

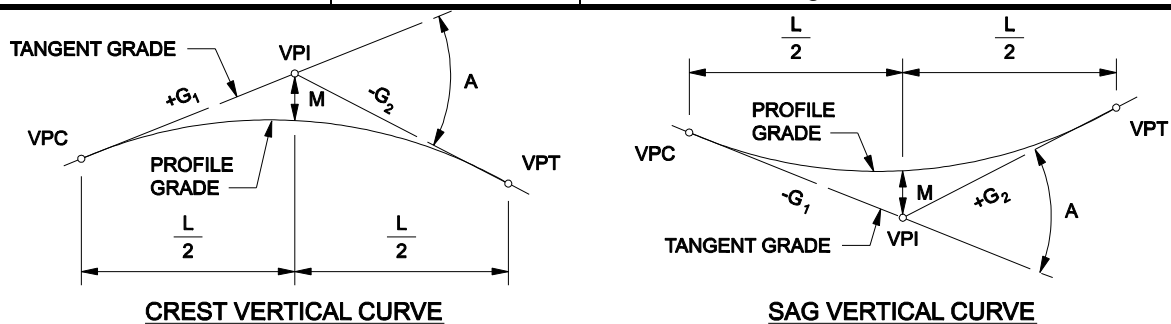
2. Minimum Length. For most sag vertical curves, the minimum length of curve should be based on Equation 10.5-3 (i.e.,  $L_{\min} = 3V$  ( $L_{\min} = 0.6V$ )). For aesthetics, the suggested minimum length of a sag vertical curve on a rural highway is 1000' (300 m).
3. Drainage. Drainage should be considered in the design of sag vertical curves where curbed sections are used. Drainage problems are minimized if the sag vertical curve is sharp enough so that a minimum longitudinal grade of at least 0.3% is reached at a point about 50' (15 m) from either side of the low point. To achieve this objective, the length of the vertical curve should be based upon a K-value of 167 (50) or less. For sag vertical curves on curbed sections where this K-value is exceeded, the drainage design should be more carefully evaluated near the low point. For example, it may be necessary to install flanking inlets on either side of the low point.

### **10.5.3 Vertical Curve Computations**

The following will apply to the mathematical design of vertical curves:

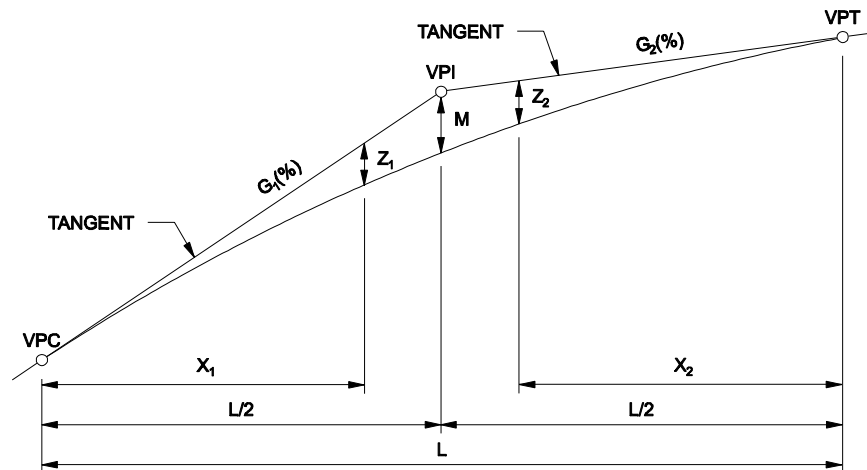
1. Definitions. Figure 10.5D presents the common terms and definitions used in vertical curve computations.
2. Measurements. All measurements for vertical curves are made on the horizontal or vertical plane, not along the profile grade. With the simple parabolic curve, the vertical offsets from the tangent vary as the square of the horizontal distance from the VPC or VPT. Elevations along the curve are calculated as proportions of the vertical offset at the point of vertical intersection (VPI). The necessary formulas for computing the vertical curve are shown in Figure 10.5E. Figure 10.5F provides an example of how to use these formulas.
3. Unsymmetrical Vertical Curve. Occasionally, it is necessary to use an unsymmetrical vertical curve to obtain clearance on a structure or to meet other field conditions. This curve is similar to the parabolic vertical curve, except the curve does not vary symmetrically about the VPI. The necessary formulas for computing the unsymmetrical vertical curve are shown in Figure 10.5G.
4. Vertical Curve Through Fixed Point. A vertical highway curve often must be designed to pass through an established point. For example, it may be necessary to tie into an existing transverse road or to clear existing structures. See Figure 10.5H. Figure 10.5I illustrates an example on how to use these formulas.

ELEMENT	ABBREVIATION	DEFINITION
Vertical Point of Curvature	VPC	The point at which a tangent grade ends and the vertical curve begins.
Vertical Point of Tangency	VPT	The point at which the vertical curve ends and the tangent grade begins.
Vertical Point of Intersection	VPI	The point where the extension of two tangent grades intersect.
Grade	$G_1, G_2$	The rate of slope between two adjacent VPI's expressed as a percent. The numerical value for percent of grade is the vertical rise or fall in feet (meters) for each 100' (100 m) of horizontal distance. Upgrades in the direction of stationing are identified as plus (+). Downgrades are identified as minus (-).
External Distance	M	The vertical distance (offset) between the VPI and the roadway surface along the vertical curve.
Algebraic Difference in Grade	A	The value of A is determined by the deflection in percent between two tangent grades ( $G_2 - G_1$ ).
Length of Vertical Curve	L	The horizontal distance in feet (meters) from the VPC to the VPT.
Tangent Elevation	Tan. Elev.	The elevation on the tangent line between the VPC and VPI and the VPI and VPT.
Elevation on Vertical Curve	Curve Elev.	The elevation of the vertical curve at any given point along the curve.
Horizontal Distance	X	Horizontal distance measured from the VPC or VPT to any point on the vertical curve, in meters.
Tangent Offset	Z	Vertical distance from the tangent line to any point on the vertical curve, in feet (meters).
Low/High Point	$X_T$	The station at the high point for crest curves or the low point for sag curves.
Symmetrical Curve	—	The VPI is located at mid-point between VPC and VPT stationing.
Unsymmetrical Curve	—	The VPI is <u>not</u> located at mid-point between VPC and VPT stationing.



### VERTICAL CURVE DEFINITIONS

Figure 10.5D



M = External distance, ft(m)

Z = Any tangent offset, ft(m)

L = Horizontal length of vertical curve, ft(m)

X = Horizontal distance from VPC or VPT to any ordinate "Z," ft(m)

G<sub>1</sub> & G<sub>2</sub> = Rates of grade, expressed algebraically, percent

NOTE: ALL EXPRESSIONS TO BE CALCULATED ALGEBRAICALLY  
(Use algebraic signs of grades; grades in percent.)

1. Elevations of VPC and VPI:

$$ELEV. OF VPC = ELEV. VPI - G_1 \left( \frac{L}{200} \right) \quad \text{(Equation 10.5-6)}$$

$$ELEV. OF VPT = ELEV. VPI + G_2 \left( \frac{L}{200} \right) \quad \text{(Equation 10.5-7)}$$

2. For the elevation of any point "X" on the vertical curve:

$$CURVE ELEV. = TAN ELEV. + Z \quad \text{(Equation 10.5-8)}$$

Where:

Left of VPI (X<sub>1</sub> measured from VPC):

$$(a) \quad TAN. ELEV. = VPC ELEV. + G_1 \left( \frac{X_1}{100} \right)$$

$$(b) \quad Z_1 = X_1^2 \frac{(G_2 - G_1)}{200 L}$$

Right of VPI (X<sub>2</sub> measured from VPT):

$$(a) \quad TAN ELEV. = VPT ELEV. - G_2 \left( \frac{X_2}{100} \right)$$

$$(b) \quad Z_2 = X_2^2 \frac{(G_2 - G_1)}{200 L}$$

3. Calculating high or low point in the vertical curve:

$$(a) \quad \text{To determine distance "X}_T\text{" from VPC:} \quad X_T = \frac{L G_1}{G_1 - G_2} \quad \text{(Equation 10.5-9)}$$

$$(b) \quad \text{To determine high or low point stationing:} \quad VPC Sta. + X_T \quad \text{(Equation 10.5-10)}$$

$$(c) \quad \text{To determine high or low point elevation on the vertical curve:} \quad ELEV. \cdot HIGHORLOW POINT = ELEV. VPC - \frac{L G_1^2}{(G_2 - G_1) 200} \quad \text{(Equation 10.5-11)}$$

### SYMMETRICAL VERTICAL CURVE EQUATIONS

Figure 10.5E

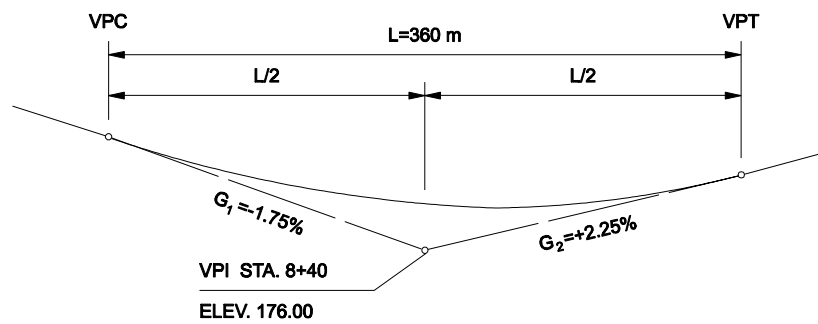


Elev. of VPI = 176.00 m  
 Station of VPI = 8+40  
 L = 360 m  
 Symmetrical Vertical Curve

**Problem:** Compute the vertical curve elevations for each 20 m station. Compute the low point elevation and stationing.

**Solution:**

1. Draw a diagram of the vertical curve and determine the station at the beginning (VPC) and the end (VPT) of the curve.



$$\text{VPC Station} = \text{VPI Sta.} - \frac{1}{2}L = (8+40) - (1+80) = 6+60$$

$$\text{VPT Station} = \text{VPI Sta.} + \frac{1}{2}L = (8+40) + (1+80) = 10+20$$

2. Vertical curve equations:

$$\text{CURVE ELEV.} = \text{TAN. ELEV.} + Z$$

(Equation 10.5-8)

Where:

Left of VPI ( $X_1$  measured from VPC):

Right of VPI ( $X_2$  measured from VPT):

$$(a) \quad \text{TAN ELEV.} = \text{VPC ELEV.} + G_1 \left( \frac{X_1}{100} \right)$$

$$(a) \quad \text{TAN ELEV.} = \text{VPT ELEV.} - G_2 \left( \frac{X_2}{100} \right)$$

$$(b) \quad Z_1 = X_1^2 \frac{(G_2 - G_1)}{200L}$$

$$(b) \quad Z_2 = X_2^2 \frac{(G_2 - G_1)}{200L}$$

3. Set up a table to show the vertical curve elevations at the 20 meter stations, substituting the values into the above equations.

### VERTICAL CURVE COMPUTATIONS (Example 10.5-1)

Figure 10.5F

**Example 10.5-1 (US Customary)**

Solution: (continued)

Station	Inf.	Tangent Elevation	X	X <sup>2</sup>	Z=X <sup>2</sup> /60000	Grade Elevation
9+00	VPC	587.930	0	0	0	587.93
9+50		587.055	50	2500	0.0417	587.10
10+00		586.180	100	10000	0.1667	586.35
10+50		585.305	150	22500	0.3750	585.68
11+00		584.430	200	40000	0.6667	585.10
11+50		583.555	250	62500	1.0417	584.60
12+00		582.680	300	90000	1.5000	584.18
12+50		581.805	350	122500	2.0417	583.85
13+00		580.930	400	160000	2.6667	583.60
13+50		580.055	450	202500	3.3750	583.43
14+00		579.180	500	250000	4.1667	583.35
14+50		578.305	550	302500	5.0417	583.35
15+00	VPI	577.430	600	360000	6.0000	583.43
15+50		578.555	550	302500	6.0417	583.60
16+00		579.680	500	250000	4.1667	583.85
16+50		580.805	450	202500	3.3750	584.18
17+00		581.930	400	160000	2.6667	584.60
17+50		583.055	350	122500	2.0417	585.10
18+00		584.180	300	90000	1.5000	585.68
18+50		585.305	250	62500	1.0417	584.35
19+00		586.430	200	40000	0.6667	587.10
19+50		587.555	150	22500	0.3750	587.93
20+00		588.680	100	10000	0.1667	588.85
20+50		589.805	50	2500	0.0417	589.85
21+00	VPT	590.930	0	0	0	590.93

1. Calculate low point (Equations 10.5-9, 10.5-10 and 10.5-11):

$$X_T = \frac{LG_1}{G_1 - G_2} = \frac{1200(-1.75)}{-1.75 - 2.25} = \frac{-630.0}{-4.00} = 525.00 \text{ feet from VPC}$$

therefore, the Station at low point is:

$$VPC_{STA} + X_T = (9+00) + (5+25) = 14+25$$

elevation of low point on curve equals:

$$Elev.VPC - \frac{LG_1^2}{(G_2 - G_1)200} = 179.15 - \frac{360(-1.75)^2}{(2.25 - (-1.75))200} = 179.15 - 1.38 = 177.77 \text{ ft}$$

**Example 10.5-1 (Metric)**

Solution: (continued)

Station	Inf.	Tangent Elevation	X	X <sup>2</sup>	Z=X <sup>2</sup> /18000	Grade Elevation
6+60	VPC	179.15	0	0	0.00	179.15
6+80		178.80	20	400	0.02	178.82
7+00		178.45	40	1 600	0.09	178.54
7+20		178.10	60	3 600	0.20	178.30
7+40		177.75	80	6 400	0.36	178.11
7+60		177.40	100	10 000	0.56	177.96
7+80		177.05	120	14 400	0.80	177.85
8+00		176.70	140	19 600	1.09	177.79
8+20		176.35	160	25 600	1.42	177.77
8+40	VPI	176.00	180	32 400	1.80	177.80
8+60		176.45	160	25 600	1.42	177.87
8+80		176.90	140	19 600	1.09	177.99
9+00		177.35	120	14 400	0.80	178.15
9+20		177.80	100	10 000	0.56	178.36
9+40		178.25	80	6 400	0.36	178.61
9+60		178.70	60	3 600	0.20	178.90
9+80		179.15	40	1 600	0.09	179.24
10+00		179.60	20	400	0.02	179.62
10+20	VPT	180.05	0	0	0.00	180.05

2. Calculate low point (Equations 10.5-9, 10.5-10 and 10.5-11):

$$X_T = \frac{LG_1}{G_1 - G_2} = \frac{360(-1.75)}{-1.75 - 2.25} = \frac{-630.0}{-4.00} = 157.5 \text{ meters from VPC}$$

therefore, the Station at low point is:

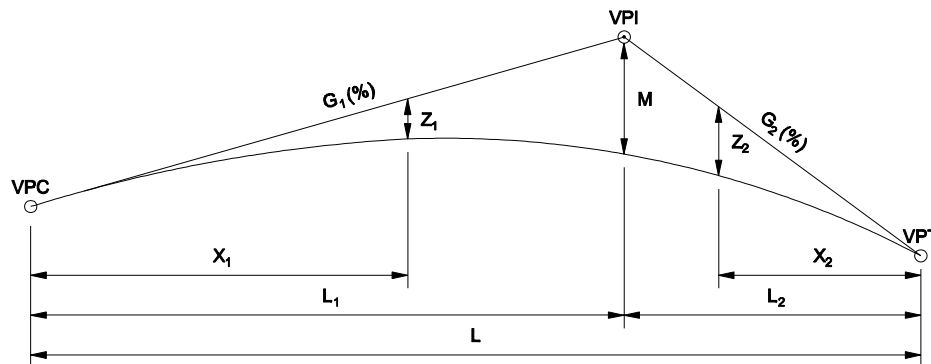
$$VPC_{STA} + X_T = (6 + 60) + (1 + 57.5) = 8 + 17.5$$

elevation of low point on curve equals:

$$Elev.VPC - \frac{LG_1^2}{(G_2 - G_1)200} = 179.15 - \frac{360(-1.75)^2}{(2.25 - (-1.75))200} = 179.15 - 1.38 = 177.77 \text{ m}$$

**VERTICAL CURVE COMPUTATIONS**  
**(Example 10.5-1)**  
**(continued)**

Figure 10.5F



$M$  = Offset from the VPI to the curve (external distance), ft (m)

$Z$  = Any tangent offset, ft (m)

$L$  = Horizontal length of vertical curve, ft (m)

$L_1$  = Horizontal distance from VPC to VPI, ft (m)

$L_2$  = Horizontal distance from VPI to VPT, ft (m)

$X$  = Horizontal distance from VPC or VPT to any ordinate " $Z$ ," ft (m)

$G_1$  &  $G_2$  = Rates of grade, expressed algebraically, percent

NOTE: ALL EXPRESSIONS TO BE CALCULATED ALGEBRAICALLY  
(Use algebraic signs of grades; grades in percent.)

1. Elevations of VPC and VPI:

$$ELEV. \text{ OF } VPC = ELEV. \text{ VPI} - G_1 \left( \frac{L_1}{100} \right) \quad (\text{Equation 10.5-12})$$

$$ELEV. \text{ OF } VPT = ELEV. \text{ VPI} + G_2 \left( \frac{L_2}{100} \right) \quad (\text{Equation 10.5-13})$$

2. For the elevation of any point " $X$ " on the vertical curve:

$$CURVE \text{ ELEV.} = TAN. \text{ ELEV.} + Z \quad (\text{Equation 10.5-14})$$

Where:

Left of VPI ( $X_1$  measured from VPC):

Right of VPI ( $X_2$  measured from VPT):

$$(a) \quad TAN. \text{ ELEV.} = VPC \text{ ELEV.} + G_1 \left( \frac{X_1}{100} \right)$$

$$(a) \quad TAN. \text{ ELEV.} = VPT \text{ ELEV.} - G_2 \left( \frac{X_2}{100} \right)$$

$$(b) \quad Z_1 = X_1^2 \left( \frac{L_2}{L_1} \right) \left( \frac{G_2 - G_1}{200 L} \right)$$

$$(b) \quad Z_2 = X_2^2 \left( \frac{L_1}{L_2} \right) \left( \frac{G_2 - G_1}{200 L} \right)$$

**UNSYMMETRICAL VERTICAL CURVE EQUATIONS**

**Figure 10.5G**

3. Calculating High or Low Point on Curve:

Note: Two answers will be determined by solving the equations below. Only one answer is correct. The incorrect answer is where  $X_T > L_1$  on the left side of the VPI or where  $X_T > L_2$  on the right side of the VPI.

- a. Assume high or low point occurs left of VPI to determine the distance,  $X_T$ , from VPC:

$$X_T = \frac{L_1}{L_2} \left[ \frac{G_1 L}{(G_1 - G_2)} \right] \quad (\text{Equation 10.5-15})$$

Note: Does  $X_T > L_1$ ? If yes, this answer is incorrect and the high or low point is on the right side of the VPI. (Go to step d. to solve for the high or low point elevation.) If no, then this is the correct answer and proceed with steps b. and c. below.)

- b. To determine high or low point stationing (where  $X_T < L_1$ ):

$$STA_{HIGH OR LOW POINT} = VPC STA. + X_T \quad (\text{Equation 10.5-16})$$

- c. To determine high or low point elevation on vertical curve (when  $X_T < L_1$ ):

$$ELEV_{HIGH OR LOW POINT} = ELEV.VPC - \frac{L_1}{L_2} \left[ \frac{LG_1^2}{(G_2 - G_1)200} \right] \quad (\text{Equation 10.5-17})$$

- d. If  $X_T > L_1$  from step a., the high or low point occurs right of the VPI. Determine the distance  $X_T$  from the VPT:

$$X_T = \frac{L_2}{L_1} \left[ \frac{G_2 L}{(G_2 - G_1)} \right] \quad (\text{Equation 10.5-18})$$

- e. To determine high or low point stationing:

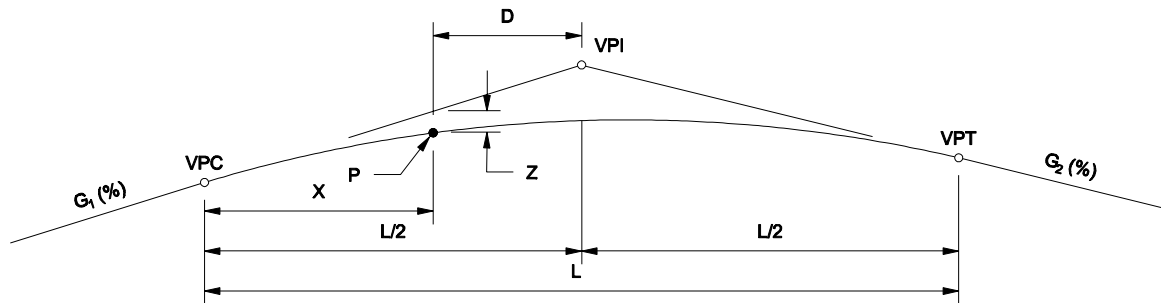
$$STA_{HIGH OR LOW POINT} = VPT STA. - X_T \quad (\text{Equation 10.5-19})$$

- f. To determine high or low point elevation on the vertical curve:

$$ELEV_{HIGH OR LOW POINT} = ELEV.VPT - \frac{L_2}{L_1} \left[ \frac{LG_2^2}{(G_2 - G_1)200} \right] \quad (\text{Equation 10.5-20})$$

### UNSYMMETRICAL VERTICAL CURVE EQUATIONS (continued)

Figure 10.5G



TO PASS A SYMMETRICAL VERTICAL CURVE THROUGH A GIVEN POINT (P)

- $G_1$  = Grade In, %                       $X$  = Distance from "P" to VPC, m  
 $G_2$  = Grade Out, %                     $D$  = Distance from "P" to VPI, m  
 $A$  = Algebraic difference in grades, %     $L$  = Length of vertical curve, m  
 $Z$  = Vertical curve correction at point "P", m

Given:         $G_1, G_2, D$

Find:        Length of vertical curve

Solution:

1.    Find algebraic difference in grades:

$$A = G_2 - G_1$$

2.    Find vertical curve correction at Point P:

From Equation 10.5-8, ( $x$  measured from VPC):

$$Z = X^2 \left( \frac{G_2 - G_1}{200L} \right)$$

3.    From inspection of the above diagram:

$$X + D = L/2 \text{ or } L = 2(X + D) \quad \text{(Equation 10.5-21)}$$

By substituting  $2(X+D)$  for  $L$ , and  $A$  for  $(G_2-G_1)$  into Equation 10.5-8. Yields:

$$AX^2 + (-400Z)X + (-400DZ) = 0 \quad \text{(Equation 10.5-22)}$$

**VERTICAL CURVE COMPUTATIONS**

**Figure 10.5H**

4. Solve for "X" given the quadratic equation:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{400Z \pm \sqrt{160\,000Z^2 + 1600ADZ}}{2A} \quad (\text{Equation 10.5-23})$$

Solving for X will result in two answers. If both answers are positive, there are two solutions. If one answer is negative, it can be eliminated and only one solution exists.

5. Substitute X and D into Equation 10.5-21 and solve for L:

Note: Two positive X values, will result in two L solutions. Desirably, the longer vertical curve solution should be used provided it meets the stopping sight distance criteria (based on the selected design speed and algebraic difference in grades, see Figures 10.5A and 10.5C).

### VERTICAL CURVE COMPUTATIONS (continued)

#### Figure 10.5H

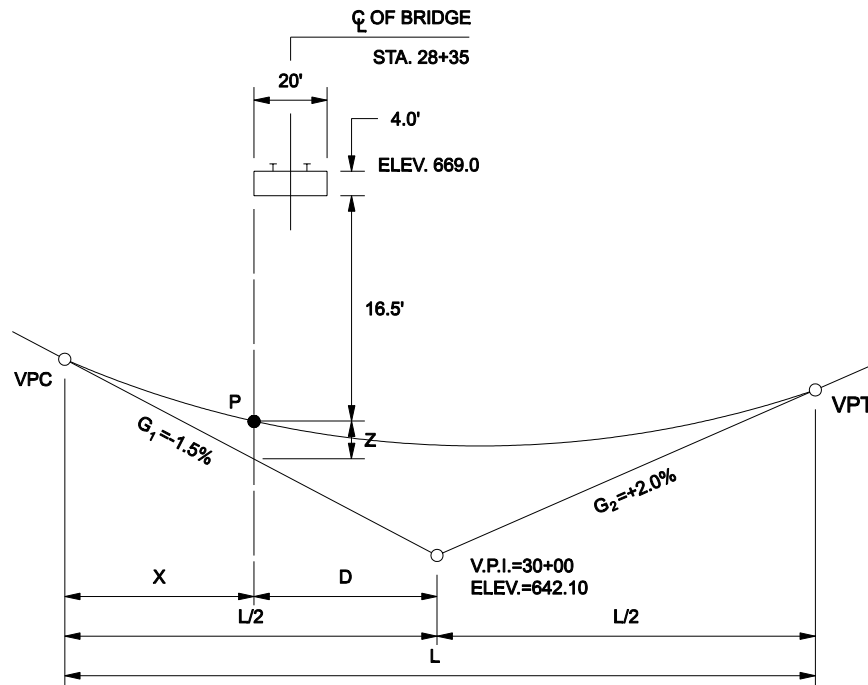
#### Example 10.5-2 (US Customary)

Given: Design Speed = 55 mph  
 $G_1 = -1.5\%$   
 $G_2 = +2.0\%$   
 $A = 3.5\%$   
 VPI Station = 30+00  
 VPI Elevation = 642.10

Problem: At Station 28+35, the new highway must pass under the center of an existing railroad which is at elevation 669.0' at the highway centerline. The railroad bridge that will be constructed over the highway will be 4' in depth, 20' in width and at right angles to the highway. What would be the length of the symmetrical vertical curve that would provide a 16.5' clearance under the railroad bridge?

Solution:

1. Sketch the problem with known information labeled.



**VERTICAL CURVE COMPUTATIONS**  
(Example 10.5-2)

**Figure 10.5I**

**Example 10.5-2** (US Customary continued)

2. Determine the station where the minimum 16.5' vertical clearance will occur (Point P):

From inspection of the sketch, the critical location is on the left side of the railroad bridge. The critical station is:

$$STA. P = BRIDGE CENTERLINE STATION - \frac{1}{2}(BRIDGE WIDTH)$$

$$STA. P = STA. 28 + 35 - \frac{20}{2}$$

$$STA. P = STA. 28 + 25$$

3. Determine the elevation of Point P:

$$ELEV. P = ELEV. TOP RAILROD BRIDGE - BRIDGE DEPTH - CLEARANCE$$

$$ELEV. P = 669.00' - 4.00' - 16.50'$$

$$ELEV. P = 648.50'$$

4. Determine distance, D, from Point P to VPI:

$$\begin{aligned} D &= STA. VPI - STA. P \\ &= (30 + 00) - (28 + 25) \\ &= 175' \end{aligned}$$

5. Determine the tangent elevation at Point P:

$$\begin{aligned} ELEV. TANGENT AT P &= ELEV. VPI - G_1 \left( \frac{D}{100} \right) \\ &= 642.10 - (-1.5) \left( \frac{175}{100} \right) \\ &= 644.73' \end{aligned}$$

6. Determine the vertical curve correction (Z) at Point P:

$$\begin{aligned} Z &= ELEV. ON CURVE - ELEV. OF TANGENT \\ &= 648.50' - 644.73' \\ &= 3.77' \end{aligned}$$

**VERTICAL CURVE COMPUTATIONS**  
**(Example 10.5-2)**  
**(continued)**  
**Figure 10.5I**

**Example 10.5-2** (US Customary continued)

7. Solve for X using Equation 10.5-23:

$$X = \frac{400Z \pm \sqrt{160000Z^2 + 1600ADZ}}{2A}$$

$$X = \frac{400(3.77) \pm \sqrt{160000(3.77)^2 + 1600(3.5)(175)(3.77)}}{2(3.5)}$$

$$X = 564.44' \quad \text{AND} \quad X = -133.58' \quad (\text{Disregard})$$

8. Using Equation 10.5-21, solve for L:

$$L = 2(X + D)$$

$$L = 2(564.44 + 175)$$

$$L = 1478.88'$$

9. Determine if the solution meets the desirable stopping sight distance for the 90 km/h design speed. From Figure 10.5C, the desirable K-value:

$$K = 96$$

The algebraic difference in grades:

$$A = G_2 - G_1 = (+2.0) - (-1.5) = 3.5$$

From Equation 10.5-5, the minimum length of vertical curve which meets the desirable stopping sight distance:

$$\begin{aligned} L_{MIN} &= KA \\ &= (96)3.5 \\ &= 336' \end{aligned}$$

L = 1478.88' will provide the desirable stopping sight distance.

(Note: This would be rounded up to 1500' for recording on the plans.)

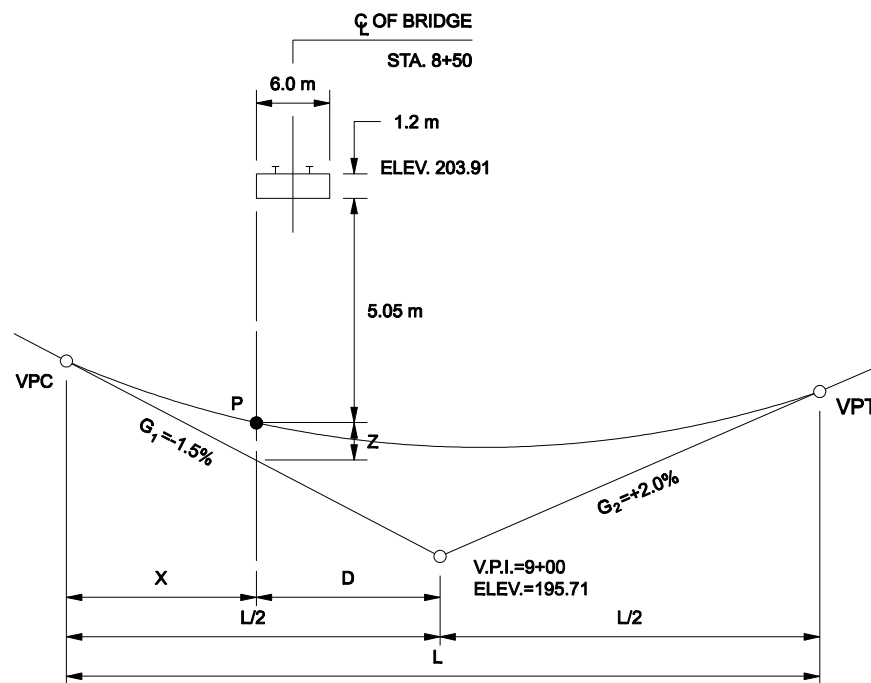
**Example 10.5-2 (Metric)**

Given: Design Speed = 90 km/h  
 $G_1 = -1.5\%$   
 $G_2 = +2.0\%$   
 $A = 3.5\%$   
VPI Station = 9+00  
VPI Elevation = 195.71

Problem: At Station 8+50, the new highway must pass under the center of an existing railroad which is at elevation 203.91 m at the highway centerline. The railroad bridge that will be constructed over the highway will be 1.2 m in depth, 6.0 m in width and at right angles to the highway. What would be the length of the symmetrical vertical curve that would provide a 5.05 m clearance under the bridge?

Solution:

1. Sketch the problem with known information labeled.



**VERTICAL CURVE COMPUTATIONS**  
**(Example 10.5-2)**

**Figure 10.5I**

**Example 10.5-2** (Metric continued)

2. Determine the station where the minimum 5.05 m vertical clearance will occur (Point P):

From inspection of the sketch, the critical location is on the left side of the railroad bridge. The critical station is:

$$STA. P = BRIDGE CENTERLINE STATION - \frac{1}{2}(BRIDGE WIDTH)$$

$$STA. P = STA. 8 + 50 - \frac{1}{2}(6 m)$$

$$STA. P = STA. 8 + 47$$

3. Determine the elevation of Point P:

$$ELEV. P = ELEV. TOP RAILROD BRIDGE - BRIDGE DEPTH - CLEARANCE$$

$$ELEV. P = 203.91 m - 1.20 m - 5.05 m$$

$$ELEV. P = 197.66 m$$

4. Determine distance, D, from Point P to VPI:

$$D = STA. VPI - STA. P$$

$$= (9 + 00) - (8 + 47)$$

$$= 53 m$$

5. Determine the tangent elevation at Point P:

$$ELEV. TANGENT AT P = ELEV. VPI - G_1 \left( \frac{D}{100} \right)$$

$$= 195.71 m - (-1.5) \left( \frac{53}{100} \right)$$

$$= 196.51 m$$

6. Determine the vertical curve correction (Z) at Point P:

$$Z = ELEV. ON CURVE - ELEV. OF TANGENT$$

$$= 197.66 - 196.51$$

$$= 1.15 m$$

**VERTICAL CURVE COMPUTATIONS**  
**(Example 10.5-2)**  
**(continued)**

**Figure 10.5I**

**Example 10.5-2** (Metric continued)

7. Solve for X using Equation 10.5-23:

$$X = \frac{400Z \pm \sqrt{160\,000Z^2 + 1600ADZ}}{2A}$$

$$X = \frac{400(1.15) \pm \sqrt{160\,000(1.15)^2 + 1600(3.5)(53)(1.15)}}{2(3.5)}$$

$$X = 171.94 \text{ m} \quad \text{AND} \quad X = -40.51 \text{ m (Disregard)}$$

8. Using Equation 10.5-21, solve for L:

$$L = 2(X + D)$$

$$L = 2(171.94 + 53)$$

$$L = 449.88 \text{ m}$$

9. Determine if the solution meets the desirable stopping sight distance for the 90 km/h design speed. From Figure 10.5C, the desirable K-value:

$$K = 41$$

The algebraic difference in grades:

$$A = G_2 - G_1 = (+2.0) - (-1.5) = 3.5$$

From Equation 10.5-5, the minimum length of vertical curve which meets the desirable stopping sight distance:

$$L_{MIN} = KA$$

$$= (41)3.5$$

$$= 143.5 \text{ m}$$

L = 449.88 m will provide the desirable stopping sight distance.

(Note: This would be rounded up to 460 meters for recording on the plans.)



## 10.6 VERTICAL CLEARANCES

Figure 10.6A summarizes the minimum vertical clearances for various highway classifications and conditions.

Type	Minimum Clearance (m)
Freeway Under	17.0' (5.20 m) (1)
Arterial Under	17.0' (5.20 m) (1)
Collector Under	16.5' (5.05 m) (1)
Roadway under Pedestrian Bridge	(2)
Roadway under Traffic Signal	17.0' (5.35 m) (1) (3)
Railroad under Roadway (Typical)	23.29' (7.10 m) (4)
Roadway under Sign Truss	17.0' (5.35 m) (1)

*Notes:*

1. *Value allows 0.50' (150 mm) for future resurfacing.*
2. *The vertical clearance should be the same as for the roadway under a highway bridge for that type of facility.*
3. *Distance is measured from roadway surface to the bottom of signal at the bottom of the back plate.*
4. *Contact the Utilities Section and the Bridge Bureau to determine the allowable railroad vertical clearance.*

### MINIMUM VERTICAL CLEARANCES

**Figure 10.6A**

